



JEEEnius AI

PREVIOUS YEAR QUESTIONS · 2019–2026

JEE Main

Chapter-wise PYQs

Mathematics**2019–2026** Questions from 8 years · plus AIEEE 2002–2012 appendix

Chapter-wise previous year questions with full solutions · Appendix:
AIEEE 2002–2012

3,278

QUESTIONS

11

FIGURES

15

CHAPTERS



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JEE Main Chapter-wise PYQs · Mathematics

Chapter-wise previous year questions with full solutions · Appendix: AIEEE 2002–2012

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How to use this book

- **Chapter-wise, newest first.** Every chapter opens with its questions, latest year first, followed by an answer key and then full solutions. Attempt the questions before turning to the key.
- **Source line.** Each question shows where it was asked — year, date and shift, and its type when it is not a single-correct MCQ.
- **Weightage table.** Mathematics opens with a table of how often every chapter was asked, overall and in the last five years. Use it to plan revision.
- **More than an answer.** Solutions carry a difficulty rating, a suggested time and the topic; many add a quick approach, the common traps students fall into, and what to revise.
- **Ref numbers.** Each solution carries a Ref number. Quote it to report an error.

Questions by year

2002 57	2003 73	2004 71	2005 69	2006 36	2007 36	2008 34	2009 29	2010 30	2011 28	2012 29	2019 442
2020 330	2021 333	2022 286	2023 423	2024 333	2025 291	2026 348					



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JEE Main 2019–2026

2,786 questions

Mathematics

Chapter	All years	Last 5 years
Sets, Relations and Functions	185	113
Complex Numbers and Quadratic Equations	181	109
Matrices and Determinants	197	115
Permutations and Combinations	159	106
Binomial Theorem and its Applications	123	85
Sequence and Series	174	98
Limit, Continuity and Differentiability	190	92
Integral Calculus	245	146
Differential Equations	140	85
Coordinate Geometry	483	292
Three Dimensional Geometry	219	141
Vector Algebra	108	73
Statistics and Probability	202	122
Trigonometry	179	104
Other Mathematics questions	1	0

MATHEMATICS · CHAPTER 1

Sets, Relations and Functions

185 questions ·

2026 13

2025 10

2024 18

2023 42

2022 30

2021 20

2020 25

2019 27

1 JEE MAIN 2026

Let $A = \{2, 3, 4, 5, 6\}$. Let R be a relation on the set $A \times A$ given by $(x, y)R(z, w)$ iff x divides z and $y \leq w$. Then the number of elements in R is _____.

- (A) 96 (B) 100
(C) 120 (D) 150

2 JEE MAIN 2026

Let $R = \{(x, y) \in N \times N : \log_e(x + y) \leq 2\}$. Then the minimum number of elements required to be added in R to make it a transitive relation is _____.

- (A) 12 (B) 13
(C) 15 (D) 16

3 JEE MAIN 2026

The number of relations defined on the set $\{a, b, c, d\}$ which are both reflexive and symmetric is equal to

- (A) 16 (B) 1024
(C) 64 (D) 256

4 JEE MAIN 2026

Let $A = \{2, 3, 5, 7, 9\}$. Let R be the relation on A defined by xRy if and only if $2x \leq 3y$. Let ℓ be the number of elements in R , and m be the minimum number of elements required to be added in R to make it a symmetric relation. Then $\ell + m$ is equal to:

- (A) 23 (B) 21
(C) 25 (D) 27

5 JEE MAIN 2026

Let $A = \{x : |x^2 - 10| < 6\}$ and $B = \{x : |x - 2| > 1\}$. Then find $B - A$.

- (A) $[-4, -2] \cup [3, 4]$ (B) $\{2, 3\}$
(C) $(-\infty, -4] \cup [-2, 1) \cup [4, \infty)$ (D) $(-\infty, 1] \cup (2, \infty)$

6 JEE MAIN 2026

Let relation R on the set $M = \{1, 2, 3, \dots, 16\}$ be given by $R = \{(x, y) : 4y = 5x - 3, x, y \in M\}$. Then the minimum number of elements required to be added in R in order to make the relation symmetric is equal to

- (A) 1 (B) 3
(C) 2 (D) 4

7 JEE MAIN 2026

The number of elements in the relation $R = \{(x, y) : 4x^2 + y^2 < 52, x, y \in \mathbb{Z}\}$ is

- (A) 89 (B) 86
(C) 77 (D) 67

8 JEE MAIN 2026

Let $A = \{-2, -1, 0, 1, 2, 3, 4\}$. Let R be a relation on A defined by xRy if and only if $2x + y \leq 2$. Let ℓ be the number of elements in R . Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations respectively. Then $\ell + m + n$ is equal to:

- (A) 33 (B) 32
(C) 34 (D) 35

9 JEE MAIN 2026

Let $A = \{0, 1, 2, \dots, 9\}$. Let R be a relation on A defined by $(x, y) \in R$ iff $|x - y|$ is a multiple of 3. Given: Statement I: $n(R) = 36$. Statement II: R is an equivalence relation. Choose the correct

- (A) Both Statement I and Statement II are incorrect
(B) Both Statement I and Statement II are correct
(C) Statement I is correct but Statement II is incorrect
(D) Statement I is incorrect but Statement II is correct

10 JEE MAIN 2026

Consider two sets $A = \{x \in \mathbb{Z} : ||x-3| - 3| \leq 1\}$ and $B = \{x : (x-2)(x-4)/(x-1) \cdot \ln(|x-2|) = 0\}$. Then the number of onto functions $f : A \rightarrow B$ is equal to

- (A) 32 (B) 62
(C) 79 (D) 81

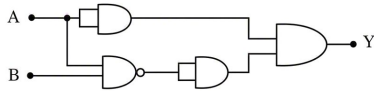
11 JEE MAIN 2026

Let R be a relation defined on the set $\{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$ by $R = \{((a, b), (c, d)) : 2a + 3b = 3c + 4d\}$. Then the number of elements in R is

- (A) 6 (B) 15
(C) 12 (D) 13

12 JEE MAIN 2026

Identify the correct truth table of the given logic circuit.



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

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(A)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(B)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(C)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

(D)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

13 JEE MAIN 2026

Given below are two statements:

Statement I: The function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x/(1+|x|)$ is one-one.

Statement II: The function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = (x^2 + 4x - 30)/(x^2 - 8x + 18)$ is many-one.

In the light of above statements, choose the correct answer from the options given below:

- (A) Both statement I and Statement II are false
 (B) Statement I is false but Statement II is true
 (C) Both Statement I and Statement II are true
 (D) Statement I is true but Statement II is false

14 JEE MAIN 2025

Let $A = \{1, 2, 3, \dots, 10\}$ and $B = \{\frac{m}{n} : m, n \in A, m < n \text{ and } \gcd(m, n) = 1\}$. Then $n(B)$ is equal to:

- (1) 36 (2) 31
 (3) 37 (4) 29

15 JEE MAIN 2025

Let $f : [0, 3] \rightarrow A$ be defined by $f(x) = 2x^3 - 15x^2 + 36x + 7$ and $g : [0, \infty) \rightarrow B$ be defined by $g(x) = \frac{x^{2025}}{x^{2025} + 1}$.

If both the functions are onto and $S = \{x \in \mathbb{Z} : x \in A \text{ or } x \in B\}$, then $n(S)$ is equal to:

- (1) 29 (2) 31
 (3) 30 (4) 36

16 JEE MAIN 2025

Let $X = \mathbb{R} \times \mathbb{R}$. Define a relation R on X as:

$(a_1, b_1)R(a_2, b_2) \Leftrightarrow b_1 = b_2$ Statement I: R is an equivalence relation. Statement II: For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y)R(a, b)\}$ represents a line parallel to $y = x$. In the light of the above statements, choose the correct answer from the options given below:

- (1) Both Statement I and Statement II are false
 (2) Statement I is true but Statement II is false
 (3) Both Statement I and Statement II are true
 (4) Statement I is false but Statement II is true

17 JEE MAIN 2025

Let A be the set of all functions $f : \mathbb{Z} \rightarrow \mathbb{Z}$ and R be a relation on A defined by $R = \{(f, g) : f(0) = g(1) \text{ and } f(1) = g(0)\}$. Then R is:

- (A) Symmetric but neither reflexive nor transitive
 (B) Reflexive but neither symmetric nor transitive
 (C) Symmetric and transitive but not reflexive
 (D) Transitive but neither reflexive nor symmetric

18 JEE MAIN 2025

If the set of all $a \in \mathbb{R} - \{1\}$, for which the roots of the equation $(1 - a)x^2 + 2(a - 3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to _____.

- (A) 7 (B) 5
 (C) 6 (D) 9

19 JEE MAIN 2025

Let $A = \{-2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by xRy iff $y = \max\{x, 1\}$. Let l be the number of elements in R . Let m and n be the minimum number of ordered pairs required to be added to R to make it reflexive and symmetric respectively. Find $l + m + n$.

- (A) 11 (B) 13
(C) 14 (D) 12

20 JEE MAIN 2025

Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by xRy iff $0 \leq x^2 + 2y \leq 4$. Let l be the number of elements in R and m be the minimum number of elements required to be added in R to make it reflexive. Then $l + m$ is equal to:

- (A) 20 (B) 17
(C) 18 (D) 19

21 JEE MAIN 2025

Consider the sets $A = \{(x, y) \in R \times R : x^2 + y^2 = 25\}$, $B = \{(x, y) \in R \times R : x^2 + 9y^2 = 144\}$, $C = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$ and $D = A \cap B$. The total number of one-one (injective) functions from the set D to the set C is:

- (A) 19320 (B) 15120
(C) 18290 (D) 17160

22 JEE MAIN 2025

Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and R be a relation on A defined by xRy iff $2x - y \in 0, 1$. Let l be the number of elements in R . Let m and n be the minimum numbers of elements required to be added in R to make it reflexive and symmetric respectively. Then $l + m + n = ?$

- (A) 18 (B) 15
(C) 17 (D) 16

23 JEE MAIN 2025 · APRIL S1

Consider two sets A and B , each containing three numbers in A.P. The sum and product of elements of A are 36 and p respectively, and the sum and product of elements of B are 36 and q respectively. Let the common differences of the APs in A and B be d and D respectively such that $D = d + 3$, $d > 0$. If $(p + q)/(p - q) = 19/5$, then $p - q$ is equal to:

- (A) 540 (B) 450
(C) 630 (D) 600

24 JEE MAIN 2024

Let the relations R_1 and R_2 on the set $X = \{1, 2, 3, \dots, 20\}$ be given by $R_1 = \{(x, y) : 2x - 3y = 2\}$ and $R_2 = \{(x, y) : -5x + 4y = 0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2 , respectively, in order to make the relations symmetric, then $M + N$ equals.

- (1) 12 (2) 16
(3) 8 (4) 10

25 JEE MAIN 2024 · NUMERICAL

Let $A = 1, 2, 3, \dots, 20$. Let R_1 and R_2 two relation on A such that $R_1 = \{(a, b) : b \text{ is divisible by } a\}$ and $R_2 = \{(a, b) : a \text{ is an integral multiple of } b\}$. Then, number of elements in $R_1 - R_2$ is equal to _____.

26 JEE MAIN 2024

Let a relation R on $N \times N$ be defined as: $(x_1, y_1)R(x_2, y_2)$ if and only if $x_1 \leq x_2$ or $y_1 \leq y_2$. Consider the two statements: (I) R is reflexive but not symmetric. (II) R is transitive. Then which one of the following is true?

- (1) Both (I) and (II) are correct. (2) Only (II) is correct.
(3) Neither (I) nor (II) is correct. (4) Only (I) is correct.

27 JEE MAIN 2024

Let $A = \{2, 3, 6, 8, 9, 11\}$ and $B = \{1, 4, 5, 10, 15\}$. Let R be a relation on $A \times B$ defined by $(a, b)R(c, d)$ if and only if $3ad - 7bc$ is an even integer. Then the relation R is:

- (1) an equivalence relation.
(2) reflexive and symmetric but not transitive.
(3) transitive but not symmetric.
(4) reflexive but not symmetric.

28 JEE MAIN 2024 · NUMERICAL

A group of 40 students appeared in an examination of 3 subjects - Mathematics, Physics & Chemistry. It was found that all students passed in at least one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, at most 11 students passed in both Mathematics and Physics, at most 15 students passed in both Physics and Chemistry, at most 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all three subjects is _____.

29 JEE MAIN 2024 · NUMERICAL

The number of symmetric relations defined on the set $\{1, 2, 3, 4\}$ which are not reflexive is _____.

30 JEE MAIN 2024

Let $A = \{1, 2, 3, \dots, 100\}$. Let R be a relation on A defined by $x, y \in R$ if and only if $2x \equiv 3y$. Let R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n . Then the minimum value of n is _____.

31 JEE MAIN 2024

The function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \frac{x^2 - 2x + 5}{x^2 - 4x + 9}$, $x \in \mathbb{R}$ is

- (A) one-one but not onto. (B) both one-one and onto.
(C) onto but not one-one. (D) neither one-one nor onto.

32 JEE MAIN 2024

If R is the smallest equivalence relation on the set $\{1, 2, 3, 4\}$ such that $\{(1, 2), (1, 3)\} \subset R$, then the number of elements in R is _____.

- (1) 10 (2) 12
(3) 8 (4) 15

33 JEE MAIN 2024

Let $A = \{n \in [100, 700] \cap \mathbb{N} : n \text{ is neither a multiple of 3 nor a multiple of 4}\}$. Then the number of elements in A is

- (1) 290 (2) 280
(3) 300 (4) 310

34 JEE MAIN 2024

Let $A = \{1, 3, 7, 9, 11\}$ and $B = \{2, 4, 5, 7, 8, 10, 12\}$. Then the total number of one-one maps $f : A \rightarrow B$, such that $f(1) + f(3) = 14$, is:

- (A) 480 (B) 240
(C) 120 (D) 180

35 JEE MAIN 2024

Let $A = \{1, 2, 3, 4, 5\}$. Let R be a relation on A defined by xRy if and only if $4x \leq 5y$. Let m be the number of elements in R and n be the minimum number of elements from $A \times A$ that are required to be added to R to make it a symmetric relation. Then $m + n$ is equal to :

- (1) 23 (2) 24
(3) 25 (4) 26

36 JEE MAIN 2024

Let $[t]$ be the greatest integer less than or equal to t . Let A be the set of all prime factors of 2310 and $f : A \rightarrow \mathbb{Z}$ be the function $f(x) = \left\lfloor \log_2 \left(x^2 + \left\lfloor \frac{x^3}{5} \right\rfloor \right) \right\rfloor$. The number of one-to-one functions from A to the range of f is

- (1) 25 (2) 24
(3) 20 (4) 120

37 JEE MAIN 2024 · NUMERICAL

Let $A = \{(x, y) : 2x + 3y = 23, x, y \in \mathbb{N}\}$ and $B = \{x : (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to _____

38 JEE MAIN 2024 · NUMERICAL

Let $A = \{2, 3, 6, 7\}$ and $B = \{4, 5, 6, 8\}$. Let R be a relation defined on $A \times B$ by $(a_1, b_1)R(a_2, b_2)$ if and only if $a_1 + a_2 = b_1 + b_2$. Then the number of elements in R is _____.

39 JEE MAIN 2024

Let R be a relation on $\mathbb{Z} \times \mathbb{Z}$ defined by $(a, b)R(c, d)$ if and only if $ad - bc$ is divisible by 5. Then R is

- (1) Reflexive and symmetric but not transitive
(2) Reflexive but neither symmetric nor transitive
(3) Reflexive, symmetric and transitive
(4) Reflexive and transitive but not symmetric

40 JEE MAIN 2024

Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B . Then the distance of the point $P(m, n)$ from the point $Q(-2, -3)$ is

- (1) 10 (2) 6
(3) 4 (4) 8

41 JEE MAIN 2024 · NUMERICAL

Let $A = \{1, 2, 3, \dots, 7\}$ and let $P(A)$ denote the power set of A . If the number of functions $f : A \rightarrow P(A)$ such that $a \in f(a), \forall a \in A$ is m^n , m and $n \in \mathbb{N}$ and m is least, then $m + n$ is equal to _____.

42 JEE MAIN 2023

An organization awarded 48 medals in event A , 25 in event B and 18 in event C . If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events?

- (1) 15 (2) 21
(3) 10 (4) 9

43 JEE MAIN 2023

Among the relations

$S = \{(a, b) : a, b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0\}$ and $T = \{(a, b) : a, b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z}\}$, which is correct?

- (1) S is transitive but T is not
(2) both S and T are symmetric
(3) neither S nor T is transitive
(4) T is symmetric but S is not

44 JEE MAIN 2023

The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to

- (1) $(\sim p) \wedge (\sim q)$ (2) $p \wedge (\sim q)$
(3) $(\sim p) \vee (\sim q)$ (4) $(\sim p) \vee q$

45 JEE MAIN 2023

The statement $B \Rightarrow ((\sim A) \vee B)$ is not equivalent to:

- (1) $B \Rightarrow (A \Rightarrow B)$
(2) $A \Rightarrow (A \Leftrightarrow B)$
(3) $A \Rightarrow ((\sim A) \Rightarrow B)$
(4) $B \Rightarrow ((\sim A) \Rightarrow B)$

46 JEE MAIN 2023

For $x \in \mathbb{R}$, two real valued functions $f(x)$ and $g(x)$ are such that, $g(x) = \sqrt{x} + 1$ and $f \circ g(x) = x + 3 - \sqrt{x}$. Then $f(0)$ is equal to

- (1) 1 (2) 5
(3) 0 (4) -3

47 JEE MAIN 2023 · NUMERICAL

Let $A = \{1, 2, 3, 4\}$ and R be a relation on the set $A \times A$ defined by $R = \{((a, b), (c, d)) : 2a + 3b = 4c + 5d\}$. Then the number of elements in R is _____.

48 JEE MAIN 2023 · NUMERICAL

For some $a, b, c \in \mathbb{N}$, let $f(x) = ax - 3$ and $g(x) = x^b + c$, $x \in \mathbb{R}$. If $(f \circ g)^{-1}(x) = \left(\frac{x-7}{2}\right)^{1/3}$, then $(f \circ g)(ac) + (g \circ f)(b)$ is equal to _____.

49 JEE MAIN 2023

Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that on the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then

- (A) the number of favourable cases of the event $(A \cup B) \cap C$ is 6
 (B) A and B are mutually exclusive
 (C) the number of favourable cases of the events A, B and C are 15, 6 and 6 respectively
 (D) B and C are independent

50 JEE MAIN 2023

The relation $R = \{a, b : \gcd(a, b) = 1, 2a \neq b, a, b \in \mathbb{Z}\}$ is:

- (1) transitive but not reflexive
 (2) symmetric but not transitive
 (3) reflexive but not symmetric
 (4) neither symmetric nor transitive

51 JEE MAIN 2023

Among the statements : $(S1) : ((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$ and $(S2) : ((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

- (1) Only $(S1)$ is a tautology
 (2) Neither $(S1)$ nor $(S2)$ is a tautology
 (3) Only $(S2)$ is a tautology
 (4) Both $(S1)$ and $(S2)$ are tautologies

52 JEE MAIN 2023

The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has:

- (1) exactly two solutions in $(-\infty, \infty)$
 (2) no solution
 (3) a unique solution in $(-\infty, 1)$
 (4) a unique solution in $(-\infty, \infty)$

53 JEE MAIN 2023 · NUMERICAL

Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $R(x, y) \in A \times A : x - y$ is odd positive integer or $x - y = 2$. The minimum number of elements that must be added to the relation R , so that it is a symmetric relation, is equal to _____.

54 JEE MAIN 2023

The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[t]$ denotes the greatest integer function, in the interval $[-1, 2]$, is :

- (1) $\frac{3}{2}$
 (2) $\frac{1}{4}$
 (3) $\frac{5}{4}$
 (4) $\frac{3}{4}$

55 JEE MAIN 2023

Let R be a relation on $\mathbb{R}$, given by $R = \{(a, b) : 3a - 3b + \sqrt{7} \text{ is an irrational number}\}$. Then R is

- (1) Reflexive but neither symmetric nor transitive
 (2) Reflexive and transitive but not symmetric
 (3) Reflexive and symmetric but not transitive
 (4) An equivalence relation

56 JEE MAIN 2023

The converse of $((\neg p) \wedge q) \Rightarrow r$ is

- (1) $((\neg p) \vee q) \Rightarrow r$
 (2) $(\neg r) \Rightarrow p \wedge q$
 (3) $(\neg r) \Rightarrow ((\neg p) \wedge q)$
 (4) $(p \vee (\neg q)) \Rightarrow (\neg r)$

57 JEE MAIN 2023

Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of $A \times B$ each having at least 3 and at most 6 elements is

- (1) 752
 (2) 782
 (3) 792
 (4) 772

58 JEE MAIN 2023

Let R be a relation defined on $\mathbb{N}$ as a Rb is $2a + 3b$ is a multiple of 5, $a, b \in \mathbb{N}$. Then R is

- (1) not reflexive
 (2) transitive but not symmetric
 (3) symmetric but not transitive
 (4) an equivalence relation

59 JEE MAIN 2023

Among the statements $(S1) : (p \Rightarrow q) \vee ((\neg p) \wedge q)$ is a tautology $(S2) : (q \Rightarrow p) \Rightarrow ((\neg p) \wedge q)$ is a contradiction

- (1) Neither $(S1)$ and $(S2)$ is True
 (2) Both $(S1)$ and $(S2)$ are True
 (3) Only $(S2)$ is True
 (4) Only $(S1)$ is True

60 JEE MAIN 2023

Let $A = \{1, 3, 4, 6, 9\}$ and $B = \{2, 4, 5, 8, 10\}$. Let R be a relation defined on $A \times B$ such that $R = \{(a_1, b_1), (a_2, b_2) : a_1 \leq b_2 \text{ and } b_1 \leq a_2\}$. Then the number of elements in the set R is

- (1) 26
 (2) 52
 (3) 160
 (4) 180

61 JEE MAIN 2023

Consider the following statements:

P : I have fever

Q : I will not take medicine

R : I will take rest

The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to:

- (A) $((\neg P) \vee \neg Q) \wedge ((\neg P) \vee R)$
 (B) $((\neg P) \vee \neg Q) \wedge ((\neg P) \vee \neg R)$
 (C) $(P \vee Q) \wedge ((\neg P) \vee R)$
 (D) $(P \vee \neg Q) \wedge (P \vee R)$

62 JEE MAIN 2023 · NUMERICAL

Let $A = \{1, 2, 3, 4, \dots, 10\}$ and $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \{(a, b) \in A \times A : 2(a - b)^2 + 3(a - b) \in B\}$ is _____.

63 JEE MAIN 2023

The number of values of $r \in \{p, q, \neg p, \neg q\}$ for which $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$ is a tautology, is:

- (1) 1 (2) 2
(3) 4 (4) 3

64 JEE MAIN 2023

The compound statement $\sim((P \wedge Q) \vee (\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to

- (1) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$
(2) $(\sim Q) \vee P$
(3) $((\sim P) \vee Q) \wedge (\sim Q)$
(4) $(\sim P) \vee Q$

65 JEE MAIN 2023 · NUMERICAL

If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to _____.

66 JEE MAIN 2023

The statement $(p \wedge (\neg q)) \Rightarrow (p \Rightarrow (\neg q))$ is

- (1) equivalent to $(\neg p) \vee (\neg q)$
(2) a tautology
(3) equivalent to $p \vee q$
(4) a contradiction

67 JEE MAIN 2023

Statement $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$ is logically equivalent to

- (1) $P \Rightarrow R \vee Q \Rightarrow R$
(2) $P \wedge R \Rightarrow Q$
(3) $P \Rightarrow R \wedge Q \Rightarrow R$
(4) $P \vee R \Rightarrow Q$

68 JEE MAIN 2023

Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $(p \rightarrow q) \Delta (p \nabla q)$ is a tautology. Then

- (1) $\Delta = \wedge, \nabla = \vee$ (2) $\Delta = \vee, \nabla = \wedge$
(3) $\Delta = \vee, \nabla = \vee$ (4) $\Delta = \wedge, \nabla = \wedge$

69 JEE MAIN 2023

In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons who speak only English is α and the number of persons who speak only Hindi is β , then the eccentricity of the ellipse $25(\beta^2 x^2 + \alpha^2 y^2) = \alpha^2 \beta^2$ is

- (1) $\sqrt{119}/12$ (2) $\sqrt{117}/12$
(3) $3\sqrt{15}/12$ (4) $\sqrt{129}/12$

70 JEE MAIN 2023

The domain of the function $f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$ is (where $[x]$ denotes the greatest integer less than or equal to x).

- (1) $(-\infty, -3) \cup (5, \infty)$ (2) $(-\infty, -2) \cup [6, \infty)$
(3) $(-\infty, -2) \cup (5, \infty)$ (4) $(-\infty, -3] \cup [6, \infty)$

71 JEE MAIN 2023

Among the two statements

$(S_1) : (p \Rightarrow q) \wedge (p \wedge (\neg q))$ is a contradiction and $(S_2) : (p \wedge q) \vee ((\neg p) \wedge q) \vee (p \wedge (\neg q)) \vee ((\neg p) \wedge (\neg q))$ is a tautology

- (1) only (S_2) is true (2) only (S_1) is true
(3) both are false (4) both are true

72 JEE MAIN 2023

Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{[x] - x}}$, where $[x]$ denotes the smallest integer greater than or equal to x . Then among the statements $(S_1) : A \cap B = (1, \infty) - \mathbb{N}$ and $(S_2) : A \cup B = (1, \infty)$

- (1) Only (S_2) is true (2) Only (S_1) is true
(3) Neither (S_1) nor (S_2) is true (4) Both (S_1) and (S_2) are true

73 JEE MAIN 2023 · NUMERICAL

Let $S = \{1, 2, 3, 5, 7, 10, 11\}$. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is _____.

74 JEE MAIN 2023

The negation of $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to

- (1) $p \vee (q \vee (\sim p))$ (2) $p \wedge q$
(3) $p \wedge (\sim q)$ (4) $p \wedge (q \wedge (\sim p))$

75 JEE MAIN 2023

The negation of the statement $((A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A$ is

- (1) equivalent to $\neg C$ (2) equivalent to $B \vee \neg C$
(3) a fallacy (4) equivalent to $\neg A$

76 JEE MAIN 2023

Let $f : \mathbb{R} - \{0, 1\} \rightarrow \mathbb{R}$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. Then $f(2)$ is equal to:

- (1) $\frac{9}{2}$ (2) $\frac{9}{4}$
(3) $\frac{7}{4}$ (4) $\frac{7}{3}$

77 JEE MAIN 2023

Negation of $(p \rightarrow q) \rightarrow (q \rightarrow p)$ is

- (A) $(p \vee \neg p)$ (B) $q \wedge (\neg p)$
(C) $(\neg q) \wedge p$ (D) $p \vee (\neg q)$

78 JEE MAIN 2023

The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is

- (A) $[\sqrt{5}, \sqrt{10}]$ (B) $[2\sqrt{2}, \sqrt{11}]$
(C) $[\sqrt{5}, \sqrt{13}]$ (D) $[\sqrt{2}, \sqrt{7}]$

79 JEE MAIN 2023

The number of functions $f : \{1, 2, 3, 4\} \rightarrow \{a \in \mathbb{Z} : |a| \leq 8\}$ satisfying $f(n) + \frac{1}{n}f(n+1) = 1, \forall n \in \{1, 2, 3\}$ is

- (1) 3 (2) 4
(3) 1 (4) 2

80 JEE MAIN 2023

Let $P(S)$ denote the power set of $S = \{1, 2, 3, \dots, 10\}$. Define the relations R_1 and R_2 on $P(S)$ as AR_1B if $(A \cap B^c) \cup (B \cap A^c) = \phi$ and AR_2B if $A \cup B^c = B \cup A^c; \forall A, B \in P(S)$. Then:

- (1) both R_1 and R_2 are equivalence relations
(2) only R_1 is an equivalence relation
(3) only R_2 is an equivalence relation
(4) both R_1 and R_2 are not equivalence relations

81 JEE MAIN 2023

The boolean expression $(\neg(p \wedge q)) \vee q$ is equivalent to

- (1) $q \rightarrow (p \wedge q)$ (2) $p \rightarrow q$
(3) $p \rightarrow (p \rightarrow q)$ (4) $p \rightarrow (p \vee q)$

82 JEE MAIN 2023 · 29 JAN S2

The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

- (A) 472 (B) 432
(C) 507 (D) 400

83 JEE MAIN 2023 · 31 JAN S1

Consider:

S1: $p \Rightarrow q \vee p \wedge \sim q$ is a tautology.

S2: $\sim p \Rightarrow \sim q \wedge \sim p \vee q$ is a contradiction.

Then

- (1) only S2 is correct (2) both S1 and S2 are correct
(3) both S1 and S2 are wrong (4) only S1 is correct

84 JEE MAIN 2022 · 24 JUN S2

Consider the following statements: A: Rishi is a judge. B: Rishi is honest. C: Rishi is not arrogant. The negation of the statement "if Rishi is a judge and he is not arrogant, then he is honest" is

- (1) $B \rightarrow (A \vee C)$
(2) $(\sim B) \wedge (A \wedge C)$
(3) $B \rightarrow ((\sim A) \vee (\sim C))$
(4) $B \rightarrow (A \wedge C)$

85 JEE MAIN 2022

Let R be a relation from the set $\{1, 2, 3, \dots, 60\}$ to itself such that $R = \{(a, b) : b = pq, \text{ where } p, q \geq 3 \text{ are prime numbers}\}$. Then, the number of elements in R is:

- (A) 600 (B) 660
(C) 540 (D) 720

86 JEE MAIN 2022

Which of the following statement is a tautology?

- (A) $((\neg q) \wedge p) \wedge q$ (B) $((\neg q) \wedge p) \wedge (p \wedge (\neg p))$
(C) $((\neg q) \wedge p) \vee (p \vee (\neg p))$ (D) $(p \wedge q) \wedge (\neg (p \wedge q))$

87 JEE MAIN 2022

Let

p : Ramesh listens to music.

q : Ramesh is out of his village

r : It is Sunday

s : It is Saturday

Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as

- (1) $((\neg q) \wedge (r \vee s)) \Rightarrow p$
(2) $(q \wedge (r \vee s)) \Rightarrow p$
(3) $p \Rightarrow (q \wedge (r \vee s))$
(4) $p \Rightarrow ((\neg q) \wedge (r \vee s))$

88 JEE MAIN 2022

Let a set $A = A_1 \cup A_2 \cup \dots \cup A_k$, where $A_i \cap A_j = \emptyset$ for $i \neq j; 1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i \text{ if and only if } x \in A_i, 1 \leq i \leq k\}$.

Then, R is:

- (1) reflexive, symmetric but not transitive
(2) reflexive, transitive but not symmetric
(3) reflexive but not symmetric and transitive
(4) an equivalence relation

89 JEE MAIN 2022

The number of choices for $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, such that $(p \Delta q) \Rightarrow ((p \Delta \neg q) \vee (\neg p \Delta q))$ is a tautology, is

- (A) 1 (B) 2
(C) 3 (D) 4

90 JEE MAIN 2022

The total number of functions, $f: \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4, 5, 6\}$ such that $f(1) + f(2) = f(3)$, is equal to

- (A) 60 (B) 90
(C) 108 (D) 126

91 JEE MAIN 2022

If the truth value of the statement $(P \wedge \neg R) \rightarrow (\neg R \wedge Q)$ is F, then the truth value of which of the following is F?

- (A) $P \vee Q \rightarrow \neg R$
(B) $R \vee Q \rightarrow \neg P$
(C) $\neg(P \vee Q) \rightarrow \neg R$
(D) $\neg(R \vee Q) \rightarrow \neg P$

92 JEE MAIN 2022

For $n \in \mathbb{N}$, let $S_n = \{z \in \mathbb{C} : |z - 3 + 2i| = \frac{n}{4}\}$ and $T_n = \{z \in \mathbb{C} : |z - 2 + 3i| = \frac{1}{n}\}$. Then the number of elements in the set $\{n \in \mathbb{N} : S_n \cap T_n = \phi\}$ is :

- (1) 0 (2) 2
(3) 3 (4) 4

93 JEE MAIN 2022

The statement $(\neg(p \iff \neg q)) \wedge q$ is:

- (1) a tautology
- (2) a contradiction
- (3) equivalent to $(p \Rightarrow q) \wedge q$
- (4) equivalent to $(p \Rightarrow q) \wedge p$

94 JEE MAIN 2022 · NUMERICAL

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A : \text{either } 1 \notin T \text{ or } 2 \in T\}$ and $C = \{T \subseteq A : \text{the sum of all the elements of } T \text{ is a prime number}\}$. Then the number of elements in the set $B \cup C$ is _____.

95 JEE MAIN 2022

Let p, q, r be three logical statements. Consider the compound statements $S_1 : ((\neg p) \vee q) \vee ((\neg p) \vee r)$ and $S_2 : p \rightarrow (q \vee r)$. Then, which of the following is NOT true?

- (1) If S_2 is True, then S_1 is True
- (2) If S_2 is False, then S_1 is False
- (3) If S_2 is False, then S_1 is True
- (4) If S_1 is False, then S_2 is False

96 JEE MAIN 2022

The statement $(p \wedge q) \Rightarrow (p \wedge r)$ is equivalent to

- (1) $q \Rightarrow (p \wedge r)$
- (2) $p \Rightarrow (p \wedge r)$
- (3) $(p \wedge r) \Rightarrow (p \wedge q)$
- (4) $(p \wedge q) \Rightarrow r$

97 JEE MAIN 2022

Let $R_1 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \leq 13\}$ and $R_2 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \neq 13\}$. Then on $\mathbb{N}$:

- (1) Both R_1 and R_2 are equivalence relations
- (2) Neither R_1 nor R_2 is an equivalence relation
- (3) R_1 is an equivalence relation but R_2 is not
- (4) R_2 is an equivalence relation but R_1 is not

98 JEE MAIN 2022

The probability that a relation R from $\{x, y\}$ to $\{x, y\}$ is both symmetric and transitive, is equal to:

- | | |
|---------------------|---------------------|
| (A) $\frac{5}{16}$ | (B) $\frac{9}{16}$ |
| (C) $\frac{11}{16}$ | (D) $\frac{13}{16}$ |

99 JEE MAIN 2022

Let a function $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined by

$$f(n) = \begin{cases} 2n, & n = 2, 4, 6, 8, \dots \\ n - 1, & n = 3, 7, 11, 15, \dots \text{ then, } f \text{ is} \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, \dots \end{cases}$$

- | | |
|------------------------------|--------------------------|
| (1) one-one but not onto | (2) onto but not one-one |
| (3) neither one-one nor onto | (4) one-one and onto |

100 JEE MAIN 2022 · NUMERICAL

Let $A = \{n \in \mathbb{N} : \text{H.C.F.}(n, 45) = 1\}$ and let $B = \{2k : k \in \{1, 2, \dots, 100\}\}$. Then the sum of all the elements of $A \cap B$ is _____.

101 JEE MAIN 2022

Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $p \nabla q \Rightarrow ((p \Delta q) \nabla r)$ is a tautology. Then $(p \nabla q) \Delta r$ is logically equivalent to :

- | | |
|-----------------------------|-----------------------------|
| (1) $(p \Delta r) \vee q$ | (2) $(p \Delta r) \wedge q$ |
| (3) $(p \wedge r) \Delta q$ | (4) $(p \nabla r) \wedge q$ |

102 JEE MAIN 2022 · NUMERICAL

Let $A = \{1, a_1, a_2, \dots, a_{18}, 77\}$ be a set of integers with $1 < a_1 < a_2 < \dots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \dots + a_{18}$ is equal to _____.

103 JEE MAIN 2022

The statement $p \Rightarrow q \vee p \Rightarrow r$ is NOT equivalent to:

- (1) $p \wedge \sim r \Rightarrow q$
- (2) $\sim q \Rightarrow \sim r \vee p$
- (3) $p \Rightarrow q \vee r$
- (4) $p \wedge \sim q \Rightarrow r$

104 JEE MAIN 2022 · NUMERICAL

Let R_1 and R_2 be relations on the set $\{1, 2, \dots, 50\}$ such that $R_1 = \{(p, p^n) : p \text{ is a prime and } n \geq 0 \text{ is an integer}\}$ and $R_2 = \{(p, p^n) : p \text{ is a prime and } n = 0 \text{ or } 1\}$. Then, the number of elements in $R_1 - R_2$ is _____.

105 JEE MAIN 2022

For $\alpha \in \mathbb{N}$, consider a relation R on $\mathbb{N}$ given by $R = \{x, y : 3x + \alpha y \text{ is a multiple of } 7\}$. The relation R is an equivalence relation if and only if

- (1) $\alpha = 14$
- (2) α is a multiple of 4
- (3) 4 is the remainder when α is divided by 10
- (4) 4 is the remainder when α is divided by 7

106 JEE MAIN 2022

Negation of the Boolean statement $(p \vee q) \Rightarrow ((\neg r) \vee p)$ is equivalent to:

- | | |
|----------------------------------|---|
| (1) $p \wedge (\neg q) \wedge r$ | (2) $(\neg p) \wedge (\neg q) \wedge r$ |
| (3) $(\neg p) \wedge q \wedge r$ | (4) $p \wedge q \wedge (\neg r)$ |

107 JEE MAIN 2022

Which of the following statements is a tautology?

- | | |
|-----------------------------------|-----------------------------------|
| (A) $\sim p \vee q \Rightarrow p$ | (B) $p \Rightarrow \sim p \vee q$ |
| (C) $\sim p \vee q \Rightarrow q$ | (D) $q \Rightarrow \sim p \vee q$ |

108 JEE MAIN 2022

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$. Then the number of elements in the set $C \subseteq A : C \cap B \neq \emptyset$ is _____.

109 JEE MAIN 2022

Let $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that $(p \wedge q) \Delta [(p \vee q) \Rightarrow q]$ is a tautology. Then Δ is equal to

- | | |
|-------------------|-----------------------|
| (1) $\wedge$ | (2) $\vee$ |
| (3) $\Rightarrow$ | (4) $\Leftrightarrow$ |

110 JEE MAIN 2022 · NUMERICAL

Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : S \times S \rightarrow S : f \text{ is onto and } f(a, b) = f(b, a) \geq a \forall (a, b) \in S \times S\}$ is _____.

111 JEE MAIN 2022 · NUMERICAL

The total number of 3-digit numbers, whose greatest common divisor with 36 is 2, is _____.

112 JEE MAIN 2022 · NUMERICAL

The sum of all the elements of the set $\{\alpha \in \{1, 2, \dots, 100\} : \text{HCF}(\alpha, 24) = 1\}$ is _____.

113 JEE MAIN 2022

$(p \wedge r) \iff (p \wedge (\neg q))$ is equivalent to $(\neg p)$ when r is

- (1) q (2) $\neg p$
(3) p (4) $\neg q$

114 JEE MAIN 2021 · 25 JUL S2

Consider function $f : A \rightarrow B$ and $g : B \rightarrow C$ such that $(g \circ f)^{-1}$ exists, then:

- (A) f and g both are one-one (B) f and g both are onto
(C) f is one-one and g is onto (D) f is onto and g is one-one

115 JEE MAIN 2021

Which of the following is the negation of the statement "for all $M > 0$, there exists $x \in S$ such that $x \geq M$ "?

- (1) there exists $M > 0$, such that $x < M$ for all $x \in S$
(2) there exists $M > 0$, there exists $x \in S$ such that $x \geq M$
(3) there exists $M > 0$, there exists $x \in S$ such that $x < M$
(4) there exists $M > 0$ such that $x \geq M$ for all $x \in S$

116 JEE MAIN 2021

Let $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$. Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to

- (1) 7 (2) 2
(3) 5 (4) 3

117 JEE MAIN 2021

Let $\mathbb{Z}$ be the set of all integers,

$$A = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + y^2 \leq 4\}$$

$$B = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 4\} \text{ and}$$

$$C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + (y-2)^2 \leq 4\}$$

If the total number of relations from $A \cap B$ to $A \cap C$ is 2^p , then the value of p is:

- (1) 25 (2) 9
(3) 16 (4) 49

118 JEE MAIN 2021

Let S_1, S_2 and S_3 be three sets defined as

$$S_1 = \{z \in \mathbb{C} : |z-1| \leq \sqrt{2}\},$$

$$S_2 = \{z \in \mathbb{C} : \text{Re}((1-i)z) \geq 1\} \text{ and}$$

$$S_3 = \{z \in \mathbb{C} : \text{Im}(z) \leq 1\}.$$

Then, the set $S_1 \cap S_2 \cap S_3$

- (1) is a singleton (2) has exactly two elements
(3) has infinitely many elements (4) has exactly three elements

119 JEE MAIN 2021

Which of the following is equivalent to the Boolean expression $p \wedge \sim q$?

- (1) $\sim p \rightarrow \sim q$ (2) $\sim (q \rightarrow p)$
(3) $\sim (p \rightarrow q)$ (4) $\sim (p \rightarrow \sim q)$

120 JEE MAIN 2021

Let $f, g : \mathbb{N} \rightarrow \mathbb{N}$ such that $f(n+1) = f(n) + f(1) \forall n \in \mathbb{N}$ and g be any arbitrary function. Which of the following statements is NOT true?

- (1) If f is onto, then $f(n) = n \forall n \in \mathbb{N}$
(2) If g is onto, then $f \circ g$ is one-one
(3) f is one-one
(4) If $f \circ g$ is one-one, then g is one-one

121 JEE MAIN 2021

Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is:

- (A) $\sim p \wedge q \wedge \sim r$ (B) $\sim p \wedge q \wedge r$
(C) $p \wedge \sim q \wedge \sim r$ (D) $p \wedge q \wedge \sim r$

122 JEE MAIN 2021

The Boolean expression $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to:

- (A) $\sim q$ (B) q
(C) p (D) $\sim p$

123 JEE MAIN 2021

Consider the statement 'The match will be played only if the weather is good and ground is not wet'. Select the correct negation from the following:

- (1) The match will not be played and weather is not good and ground is wet.
(2) If the match will not be played, then either weather is not good or ground is wet.
(3) The match will be played and weather is not good or ground is wet.
(4) The match will not be played or weather is good and ground is not wet.

124 JEE MAIN 2021 · NUMERICAL

Let $S = \{1, 2, 3, 4, 5, 6, 9\}$. Then the number of elements in the set $T = \{A \subseteq S : A \neq \phi \text{ and the sum of all the elements of } A \text{ is not a multiple of } 3\}$ is _____.

125 JEE MAIN 2021

The negation of the statement $\neg p \wedge (p \vee q)$ is:

- (1) $\neg p \vee q$ (2) $\neg p \wedge q$
(3) $p \vee \neg q$ (4) $p \wedge \neg q$

126 JEE MAIN 2021

The Boolean expression $(p \wedge \neg q) \Rightarrow (q \vee \neg p)$ is equivalent to:

- (1) $q \Rightarrow p$ (2) $p \Rightarrow q$
(3) $q \Rightarrow p$ (4) $p \Rightarrow \neg q$

127 JEE MAIN 2021

The number of elements in the set $\{x \in \mathbb{R} : (|x| - 3)|x + 4| = 6\}$ is equal to :

- (1) 3 (2) 2
(3) 4 (4) 1

128 JEE MAIN 2021

Let $A = \{2, 3, 4, 5, \dots, 30\}$ and $\sim$ be an equivalence relation on $A \times A$, defined by $(a, b) \sim (c, d)$, if and only if $ad = bc$. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair $(4, 3)$ is equal to:

- (1) 5 (2) 6
(3) 8 (4) 7

129 JEE MAIN 2021

Let $\mathbb{N}$ be the set of natural numbers and a relation R on $\mathbb{N}$ be defined by

$$R = \{(x, y) \in \mathbb{N} \times \mathbb{N} : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}.$$

Then the relation R is

- (1) symmetric but neither reflexive nor transitive
(2) reflexive but neither symmetric nor transitive
(3) reflexive and symmetric, but not transitive
(4) an equivalence relation

130 JEE MAIN 2021

Which of the following Boolean expression is a tautology?

- (A) $(p \wedge q) \vee (p \vee q)$ (B) $(p \wedge q) \vee (p \rightarrow q)$
(C) $(p \wedge q) \wedge (p \rightarrow q)$ (D) $(p \wedge q) \rightarrow (p \rightarrow q)$

131 JEE MAIN 2021 · NUMERICAL

Let $A = \{n \in \mathbb{N} \mid n^2 \leq n + 10,000\}$, $B = \{3k + 1 \mid k \in \mathbb{N}\}$ and $C = \{2k \mid k \in \mathbb{N}\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to _____.

132 JEE MAIN 2021

If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions: $f+g$, $f-g$, f/g , g/f , $g-f$, where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x) = \frac{f(x)}{g(x)}$?

- (1) $0 \leq x \leq 1$ (2) $0 \leq x < 1$
(3) $0 < x < 1$ (4) $0 < x \leq 1$

133 JEE MAIN 2021 · NUMERICAL

Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. Then the number of possible functions $f : S \rightarrow S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$, is equal to _____.

134 JEE MAIN 2020

Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\neg q \vee r)$ is F. Then the truth values of p, q, r are respectively:

- (1) T, F, T (2) F, T, F
(3) T, T, F (4) T, T, T

135 JEE MAIN 2020

Which of the following is tautology -

- (1) $\sim(p \vee \sim q) \rightarrow (p \vee q)$ (2) $(\sim p \vee q) \rightarrow (p \vee q)$
(3) $\sim(p \wedge \sim q) \rightarrow (p \vee q)$ (4) $\sim(p \vee \sim q) \rightarrow (p \wedge q)$

136 JEE MAIN 2020

Which of the following is tautology?

- (A) $(p \wedge (p \rightarrow q)) \rightarrow q$
(B) $q \rightarrow p \wedge (p \rightarrow q)$
(C) $p \vee (p \wedge q)$
(D) $(p \wedge (p \vee q))$

137 JEE MAIN 2020

The contrapositive of the statement "If I reach the station in time, then I will catch the train" is:

- (1) If I will catch the train, then I reach the station in time.
(2) If I do not reach the station in time, then I will not catch the train.
(3) If I will not catch the train, then I do not reach the station in time.
(4) If I do not reach the station in time, then I will catch the train.

138 JEE MAIN 2020

The negation of ' $\sqrt{5}$ is an integer or 5 is an irrational number' is

- (1) $\sqrt{5}$ is an integer and 5 is not an irrational Number
(2) $\sqrt{5}$ is not an integer and 5 is an irrational Number
(3) $\sqrt{5}$ is not an integer or 5 is not an irrational Number
(4) $\sqrt{5}$ is not an integer and 5 is not an irrational Number

139 JEE MAIN 2020

Given the following two statements:

$(S_1) : (q \vee p) \rightarrow (p \leftrightarrow \neg q)$ is a tautology.

$(S_2) : \neg q \wedge (\neg p \leftrightarrow q)$ is a fallacy.

Then:

- (1) only (S_1) is correct.
(2) both (S_1) and (S_2) are correct.
(3) both (S_1) and (S_2) are not correct.
(4) only (S_2) is correct.

140 JEE MAIN 2020 · NUMERICAL

Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of $m \cdot n$ is _____.

141 JEE MAIN 2020

Consider the statement: "For an integer n , if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is:

- (1) For an integer n , if $n^3 - 1$ is not even, then n is not odd.
 (2) For an integer n , if n is even, then $n^3 - 1$ is odd.
 (3) For an integer n , if n is odd, then $n^3 - 1$ is even.
 (4) For an integer n , if n is even, then $n^3 - 1$ is even.

142 JEE MAIN 2020

A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

- (1) 63 (2) 38
 (3) 54 (4) 36

143 JEE MAIN 2020

If $A = \{x \in \mathbb{R} : |x| < 2\}$ and $B = \{x \in \mathbb{R} : |x - 2| \geq 3\}$; then:

- (1) $A \cap B = (-2, -1)$ (2) $B - A = \mathbb{R} - (-2, 5)$
 (3) $A \cup B = \mathbb{R} - (2, 5)$ (4) $A - B = [-1, 2)$

144 JEE MAIN 2020

Which of the following is a tautology?

- (1) $(\neg p) \wedge (p \vee q) \rightarrow q$ (2) $(q \rightarrow p) \vee \neg(p \rightarrow q)$
 (3) $(p \rightarrow q) \wedge (q \rightarrow p)$ (4) $(\neg q) \vee (p \wedge q) \rightarrow q$

145 JEE MAIN 2020

Let $\bigcup_{i=1}^{50} X_i = \bigcup_{j=1}^n Y_j = T$, where each X_i contains 10 elements and each Y_j contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i 's and exactly 6 of sets Y_j 's, then n is equal to:

- (1) 45 (2) 15
 (3) 50 (4) 30

146 JEE MAIN 2020

Let R_1 and R_2 be two relations defined as follows:

$$R_1 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \in \mathbb{Q}\} \text{ and}$$

$$R_2 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \notin \mathbb{Q}\},$$

where $\mathbb{Q}$ is the set of all rational numbers. Then:

- (1) R_2 is transitive but R_1 is not transitive
 (2) R_1 is transitive but R_2 is not transitive
 (3) R_1 and R_2 are both transitive
 (4) Neither R_1 nor R_2 is transitive

147 JEE MAIN 2020

Solution set of $3^{3^x-1} + 2 = |3^x - 1| + |3^x - 2|$ contains

- (1) singleton set (2) two elements
 (3) at least four elements (4) infinite elements

148 JEE MAIN 2020

Let $X = \{x : 1 \leq x \leq 50, x \in \mathbb{N}\}$, $A = \{x : x \text{ is multiple of } 2\}$, $B = \{x : x \text{ is multiple of } 7\}$. Then find the number of elements in the smallest subset of X which contain elements of both A and B .

- (A) 25 (B) 7
 (C) 3 (D) 29

149 JEE MAIN 2020

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function which satisfies $f(x + y) = f(x) + f(y) \forall x, y \in \mathbb{R}$. If $f(1) = 2$ and $g(n) = \sum_{k=1}^{n-1} f(k)$, $n \in \mathbb{N}$, then the value of n , for which $g(n) = 20$, is:

- (1) 5 (2) 9
 (3) 20 (4) 4

150 JEE MAIN 2020

If $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers $\mathbb{Z}$, then the domain of R^{-1} is:

- (1) $\{-2, -1, 1, 2\}$ (2) $\{-1, 0, 1\}$
 (3) $\{-2, -1, 0, 1, 2\}$ (4) $\{0, 1\}$

151 JEE MAIN 2020

If $g(x) = x^2 + x - 1$ and $g(f(x)) = 4x^2 - 10x + 5$, then find $f\left(\frac{5}{4}\right)$.

- (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$
 (C) $-\frac{1}{3}$ (D) $\frac{1}{3}$

152 JEE MAIN 2020

Let $[t]$ denote the greatest integer $\leq t$. Then the equation in x , $[x]^2 + 2[x + 2] - 7 = 0$ has:

- (1) no integral solution (2) exactly four integral solutions
 (3) exactly two solutions (4) infinitely many solutions

153 JEE MAIN 2020

Let $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$ then inverse of $f(x)$ is

- (A) $\frac{1}{4} \log_8 \left(\frac{1+x}{1-x} \right)$
 (B) $\frac{1}{2} \log_8 \left(\frac{1-x}{1+x} \right)$
 (C) $\frac{1}{4} \log_8 \left(\frac{1-x}{1+x} \right)$
 (D) $\frac{1}{2} \log_8 \left(\frac{1+x}{1-x} \right)$

154 JEE MAIN 2020

The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to:

- (1) $\sim p \vee \sim q$ (2) $p \vee q$
 (3) $\sim p \wedge \sim q$ (4) $p \wedge \sim q$

155 JEE MAIN 2020

If $p \rightarrow (p \wedge \sim q)$ is false. Truth value of $p \wedge q$ will be

- (1) TT (2) TF
 (3) FT (4) FF

156 JEE MAIN 2020

$(p \rightarrow q) \wedge (q \rightarrow \sim p)$ is equivalent to

- (1) $\sim p$ (2) p
(3) $p \wedge q$ (4) $p \vee q$

157 JEE MAIN 2020

Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^n Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i 's and exactly 6 of sets Y_i 's, then n is equal to:

- (1) 45 (2) 15
(3) 50 (4) 30

158 JEE MAIN 2020

A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If $x\%$ of the people read both the newspapers, then a possible value of x can be:

- (1) 65 (2) 37
(3) 29 (4) 55

159 JEE MAIN 2019 · 10 JAN S1

In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is:

- (1) 42 (2) 1
(3) 38 (4) 102

160 JEE MAIN 2019

If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

- (1) $(p \vee r) \rightarrow (p \wedge r)$
(2) $(p \wedge r) \rightarrow (p \vee r)$
(3) $p \wedge r$
(4) $p \vee r$

161 JEE MAIN 2019

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in \mathbb{R}$. Then the range of f is

- (1) $[-\frac{1}{2}, \frac{1}{2}]$
(2) $\mathbb{R} - [-1, 1]$
(3) $\mathbb{R} - [-\frac{1}{2}, \frac{1}{2}]$
(4) $(-1, 1) - \{0\}$

162 JEE MAIN 2019

The contrapositive of the statement 'If you are born in India, then you are a citizen of India', is

- (1) If you are not born in India, then you are not a citizen of India.
(2) If you are a citizen of India, then you are born in India.
(3) If you are born in India, then you are not a citizen of India.
(4) If you are not a citizen of India, then you are not born in India.

163 JEE MAIN 2019

Let $\mathbb{N}$ be the set of natural numbers and two functions f and g be defined as $f, g: \mathbb{N} \rightarrow \mathbb{N}$ such that

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

and $g(n) = n - (-1)^n$. Then $f \circ g$ is:

- (1) onto but not one-one (2) Both one-one and onto
(3) One-one but not onto (4) Neither one-one nor onto

164 JEE MAIN 2019

If the function $f: \mathbb{R} \setminus \{1, -1\} \rightarrow A$ defined by $f(x) = \frac{x^2}{1-x^2}$ is surjective, then A is equal to

- (1) $[0, \infty)$ (2) $\mathbb{R} \setminus \{-1\}$
(3) $\mathbb{R} \setminus [-1, 0)$ (4) $\mathbb{R} \setminus (-1, 0)$

165 JEE MAIN 2019

Which one of the following statements is not a tautology?

- (A) $p \vee q \rightarrow p \vee (\sim q)$
(B) $p \wedge q \rightarrow (\sim p \vee q)$
(C) $p \rightarrow p \vee q$
(D) $p \wedge q \rightarrow p$

166 JEE MAIN 2019

Let $f(x) = x^2$, $x \in \mathbb{R}$. For any $A \subseteq \mathbb{R}$, define $g(A) = \{x \in \mathbb{R} : f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true?

- (1) $g(f(S)) \neq S$ (2) $f(g(S)) \neq f(S)$
(3) $f(g(S)) = S$ (4) $g(f(S)) = g(S)$

167 JEE MAIN 2019

For $x \in \mathbb{R} - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then $J(x)$ is equal to:

- (1) $f_3(x)$ (2) $\frac{1}{x} f_3(x)$
(3) $f_1(x)$ (4) $f_2(x)$

168 JEE MAIN 2019

Which one of the following Boolean expressions is a tautology?

- (A) $(p \vee q) \wedge (\sim p \vee \sim q)$
 (B) $(p \wedge q) \vee (p \wedge \sim q)$
 (C) $(p \vee q) \wedge (p \vee \sim q)$
 (D) $(p \vee q) \vee (\sim p \vee \sim q)$

169 JEE MAIN 2019

Let $S = \{1, 2, 3, \dots, 100\}$, then number of non-empty subsets A of S such that the product of elements in A is even is:

- (A) $2^{100} - 1$ (B) $2^{50} + 1$
 (C) $2^{50}(2^{50} - 1)$ (D) $2^{50} - 1$

170 JEE MAIN 2019

Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B . Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is:

- (1) 13.5 (2) 12.8
 (3) 13.9 (4) 13

171 JEE MAIN 2019

For $x \in (0, \frac{\pi}{2})$, let $fx = \sqrt{x}$, $gx = \tan x$ and $hx = \frac{1-x^2}{1+x^2}$. If $\phi x = (h \circ f) \circ g(x)$, then $\phi \frac{\pi}{3}$ is equal to:

- (1) $\tan \frac{\pi}{12}$ (2) $\tan \frac{5\pi}{12}$
 (3) $\tan \frac{7\pi}{12}$ (4) $\tan \frac{11\pi}{12}$

172 JEE MAIN 2019

Consider the following three statements:

P: 5 is a prime number
 Q: 7 is a factor of 192
 R: LCM of 5 and 7 is 35

Then the truth value of which one of the following statements is true?

- (1) $P \vee (\neg Q \wedge R)$ (2) $(P \wedge Q) \vee (\neg R)$
 (3) $(\neg P) \vee (Q \wedge R)$ (4) $(\neg P) \wedge (\neg Q \wedge R)$

173 JEE MAIN 2019

Let a function $f : (0, \infty) \rightarrow (0, \infty)$ be defined by $f(x) = |1 - \frac{1}{x}|$. Then f is:

- (A) not injective but it is surjective
 (B) injective only
 (C) neither injective nor surjective
 (D) both injective as well as surjective

174 JEE MAIN 2019

Let $A = \{x \in \mathbb{R} : x \text{ is not a positive integer}\}$. Define a function $f : A \rightarrow \mathbb{R}$ as $f(x) = \frac{2x}{x-1}$, then f is:

- (1) Injective but not surjective (2) Not injective
 (3) Surjective but not injective (4) Neither injective nor surjective

175 JEE MAIN 2019

Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall x \in \mathbb{R}$. Then the set of all $x \in \mathbb{R}$, where the function $h(x) = (f \circ g)(x)$ is increasing, is:

- (1) $[-1, -\frac{1}{2}] \cup [\frac{1}{2}, \infty)$
 (2) $[0, \infty)$
 (3) $[0, \frac{1}{2}] \cup [1, \infty)$
 (4) $[-\frac{1}{2}, 0] \cup [1, \infty)$

176 JEE MAIN 2019

Contrapositive of the statement "If two numbers are not equal, then their squares are not equal." is:

- (1) If the squares of two numbers are not equal, then the numbers are equal.
 (2) If the squares of two numbers are equal, then the numbers are not equal.
 (3) If the squares of two numbers are equal, then the numbers are equal.
 (4) If the squares of two numbers are not equal, then the numbers are not equal.

177 JEE MAIN 2019

The negation of the Boolean expression $\sim s \vee \sim r \wedge s$ is equivalent to

- (A) r (B) $s \wedge r$
 (C) $s \vee r$ (D) $\sim s \wedge \sim r$

178 JEE MAIN 2019

For any two statement p and q , the negative of the expression $p \vee (\sim p \wedge q)$ is

- (A) $\sim p \vee \sim q$ (B) $p \wedge q$
 (C) $\sim p \wedge \sim q$ (D) $p \leftrightarrow q$

179 JEE MAIN 2019

If the truth value of the statement $p \rightarrow \sim q \vee r$ is false F , then the truth values of the statements p, q, r are respectively

- (1) T, F, T (2) T, F, F
 (3) T, T, F (4) F, T, T

180 JEE MAIN 2019

Let A, B , and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?

- (1) $B \cap C \neq \phi$
 (2) $(C \cup A) \cap (C \cup B) = C$
 (3) If $(A - B) \subseteq C$, then $A \subseteq C$
 (4) If $(A - C) \subseteq B$, then $A \subseteq B$

Relations on Cartesian Product / Counting ordered pairs

Count independently for (x, z) and (y, w) . For x divides z : $x=2 \rightarrow z \in \{2, 4, 6\}$ (3 choices), $x=3 \rightarrow \{3, 6\}$ (2), $x=4 \rightarrow \{4\}$ (1), $x=5 \rightarrow \{5\}$ (1), $x=6 \rightarrow \{6\}$ (1). Total = $3+2+1+1+1 = 8$. For $y \leq w$: for $y=2 \rightarrow 5$ choices, $y=3 \rightarrow 4$, $y=4 \rightarrow 3$, $y=5 \rightarrow 2$, $y=6 \rightarrow 1$. Total = $5+4+3+2+1 = 15$. Relation size = $8 \times 15 = 120$.

COMMON TRAPS

- **Counting only elements of $A \times A$ instead of elements of the relation R .** An element of R is of the form $((x, y), (z, w))$, where $(x, y) \in A \times A$ and $(z, w) \in A \times A$. So the universe has 25×25 possible ordered pairs before conditions. *Avoid it:* Rewrite every element as $((x, y), (z, w))$ and note four variables are being chosen, not two.
- **Treating the two conditions as dependent when they are independent.** The number of valid quadruples equals $(\# \text{ valid } (x, z) \text{ pairs}) \times (\# \text{ valid } (y, w) \text{ pairs}) = 8 \times 15$ because choices are independent. *Avoid it:* Check whether conditions share variables. If no variable is shared between two conditions, multiply the separate counts.

REVISE The distinction between a relation on S and the set S itself: $R \subseteq S \times S$. · Practice translating "a divides b" as "b is a multiple of a".

2 Answer: C

Ref 12872

Hard ~2 min Transitive relation

QUICK APPROACH Start by interpreting the relation R as a set of ordered pairs based on the inequality, then think about transitive closure in terms of graph theory where each pair represents an edge in a directed graph.

Condition $\log_e(x + y) \leq 2$ implies $x + y \leq e^2$. Since $x+y$ is an integer, $x+y \leq 7$.

Thus $R = \{(x, y) \in N \times N : x+y \leq 7\}$. Assuming $N = \{1, 2, \dots\}$, possible sums $s = 2, 3, 4, 5, 6, 7$. Number of pairs with sum s is $s-1$. So $|R| = 1+2+3+4+5+6 = 21$.

All first and second coordinates that appear are in $\{1, 2, 3, 4, 5, 6\}$. View R as a directed graph on vertices $1..6$ with an edge $a \rightarrow b$ when $a+b \leq 7$. Note every vertex k has edge $k \rightarrow 1$ (since $k+1 \leq 7$ for $k \leq 6$), and $1 \rightarrow k$ for all k (since $1+k \leq 7$). Hence every vertex $1..6$ is mutually reachable (single strongly connected component). The transitive closure therefore contains every ordered pair among $\{1, \dots, 6\}$, i.e. $6 \times 6 = 36$ pairs.

Minimum elements to add = $36 - 21 = 15$.

COMMON TRAPS

- **Rounding e^2 incorrectly to 8 instead of using integer bound 7.** Since " $x, y \in N$ " makes $x+y$ an integer and $e^2 \approx 7.389$, the largest possible integer value is 7, not 8. *Avoid it:* After exponentiating, write the decimal bound and take floor for $\leq$ inequalities: $x+y \leq 7.389 \Rightarrow x+y \leq 7$.
- **Counting only original relation size instead of added elements.** The requested quantity is "minimum number of elements required to be added in R ", which is $|\text{transitive closure}| - |R|$. *Avoid it:* Underline "added" and always subtract existing ordered pairs from the closure size.

REVISE Integer bounds after applying monotonic functions such as $\log$ /exponential. · Practice counting ordered pairs under $x+y \leq n$ over positive integers.

3 Answer: C

Ref 11107

Medium ~1 min Reflexive and Symmetric Relations

QUICK APPROACH Start by applying the definitions of reflexive and symmetric relations to identify fixed and variable pairs in the relation on the set.

A relation on $A = \{a, b, c, d\}$ is a subset of $A \times A$, which has 16 ordered pairs.

Step 1. Reflexive. All 4 diagonal pairs $(a, a), (b, b), (c, c), (d, d)$ must be included. There is no choice here.

Step 2. Symmetric. The 12 off-diagonal pairs fall into $\binom{4}{2} = 6$ mirror pairs $\{(x, y), (y, x)\}$. Symmetry means each mirror pair is either fully in or fully out. That is 2 choices per pair.

Step 3. Count.

$$2^6 = 64$$

In general, a set with n elements has $2^{n(n-1)/2}$ relations that are both reflexive and symmetric.

Answer: (C) 64

COMMON TRAPS

- **Using total subset count of $A \times A$ instead of applying both conditions.** A relation on a 4-element set is a subset of 16 ordered pairs only before imposing conditions; reflexive and symmetric greatly reduce the choices. *Avoid it:* Underline condition words and ask how many pairs are forced and how many are linked before using 2^k .
- **Treating each ordered off-diagonal pair independently despite symmetry.** For a symmetric relation, (a, b) and (b, a) are not separate choices; the pair-block has only two valid states: both included or both excluded. *Avoid it:* Draw the 4×4 matrix and mirror the upper triangle to the lower triangle; only upper-triangular off-diagonal entries are free.

REVISE Relation counting using a 4×4 matrix: diagonal fixed for reflexive, upper triangle free for symmetric. · Memorize with meaning: reflexive+symmetric on n elements gives $2^{n(n-1)/2}$, while symmetric only gives $2^{n(n+1)/2}$.

4 Answer: C

Ref 11199

Hard ~2 min Types of Relations (Symmetric Relation)

QUICK APPROACH Start by listing all ordered pairs (x, y) in R that satisfy $2x \leq 3y$ to find the total number of elements ℓ using the definition of the given relation.

Step 1. Count R . $xRy \iff y \geq \frac{2x}{3}$. Go through each $x \in A = \{2, 3, 5, 7, 9\}$:

- $x = 2$: $y \geq 1.33$, all 5 values
- $x = 3$: $y \geq 2$, all 5 values
- $x = 5$: $y \geq 3.33$, so $y \in \{5, 7, 9\}$: 3
- $x = 7$: $y \geq 4.67$, so $y \in \{5, 7, 9\}$: 3
- $x = 9$: $y \geq 6$, so $y \in \{7, 9\}$: 2

So $\ell = 5 + 5 + 3 + 3 + 2 = 18$.

Step 2. Pairs with $x \neq y$. If $x < y$ then $2x < 3y$ always, so all $\binom{5}{2} = 10$ such pairs are in R . Among pairs with $x > y$, only

(3, 2), (7, 5) and (9, 7) satisfy $2x \leq 3y$. (Check: $5 + 10 + 3 = 18$.)

Step 3. Make it symmetric. For each of the 10 pairs (x, y) with $x < y$, the reverse (y, x) must be present. Only 3 reverses are already in R , so $m = 10 - 3 = 7$.

$$\ell + m = 18 + 7 = 25$$

Answer: (C) 25

COMMON TRAPS

- **Counting unordered pairs instead of ordered pairs.** A relation on A consists of ordered pairs (x, y) . The first coordinate is x and the second is y , and the condition $2x \leq 3y$ is not symmetric in x and y . *Avoid it:* Make a 5×5 table with rows x and columns y ; mark every cell satisfying $2x \leq 3y$.
- **Reversing the inequality when solving for y .** Since 3 is positive, dividing $2x \leq 3y$ by 3 gives $2x/3 \leq y$, i.e. $y \geq 2x/3$. *Avoid it:* After rearranging, test one obvious large y -value such as $y=9$; it should satisfy for all listed x because $3y$ is large.

REVISE Relation as a subset of $A \times A$ and practice ordered-pair enumeration using tables. · Use a row-column matrix to separate the count of ℓ from the missing mirror entries used for m .

5 Answer: C

Ref 11203

Medium ~2 min Set Operations and Absolute Value Inequalities

First find set A .

$$|x^2 - 10| < 6 \Rightarrow -6 < x^2 - 10 < 6 \Rightarrow 4 < x^2 < 16 \Rightarrow 2 < |x| < 4$$

Thus

$$A = (-4, -2) \cup (2, 4).$$

Now find set B .

$$\begin{aligned} |x - 2| &> 1 \\ \Rightarrow x - 2 > 1 \text{ OR } x - 2 < -1 \\ \Rightarrow x > 3 \text{ OR } x < 1 \end{aligned}$$

Thus

$$B = (-\infty, 1) \cup (3, \infty).$$

Now compute $B - A$ (elements in B but not in A).

From $(-\infty, 1)$, remove the part overlapping with A which is $(-4, -2)$.

Remaining: $(-\infty, -4] \cup [-2, 1)$.

From $(3, \infty)$, remove overlap with A which is $(3, 4)$.

Remaining: $[4, \infty)$.

Therefore

$$B - A = (-\infty, -4] \cup [-2, 1) \cup [4, \infty).$$

COMMON TRAPS

- **Using the '<' absolute-value rule for the '>' inequality in B.** For $|u| > a$, split into $u > a$ OR $u < -a$. It is an outside interval, not an inside interval. *Avoid it:* Circle the sign before splitting: '<' means AND between bounds; '>' means OR outside the bounds.
- **Forgetting the lower bound in A.** $|u| < a$ means $-a < u < a$; both lower and upper bounds are required. *Avoid it:* After removing absolute value, count two inequality signs for a '<' absolute value inequality.

REVISE Memorize and apply $|u| < a$ as an AND interval and $|u| > a$ as an OR union. · For quadratic inequalities, convert to $|x|$ before splitting into positive and negative intervals.

6 Answer: C

Ref 11277

Medium ~1.5 min Types of Relations (Symmetric Relation)

Given relation: $4y = 5x - 3$ with $x, y \in M = \{1, 2, \dots, 16\}$. First express y in terms of x : $y = (5x - 3)/4$. For (x, y) to belong to $M \times M$, y must be an integer between 1 and 16. Checking values of x from 1 to 16 where $(5x - 3)$ is divisible by 4: $x = 3 \rightarrow y = (15 - 3)/4 = 3 \rightarrow (3, 3)$; $x = 7 \rightarrow y = (35 - 3)/4 = 8 \rightarrow (7, 8)$; $x = 11 \rightarrow y = (55 - 3)/4 = 13 \rightarrow (11, 13)$. Thus $R = \{(3, 3), (7, 8), (11, 13)\}$. A relation is symmetric if $(x, y) \in R \Rightarrow (y, x) \in R$. Pair $(3, 3)$ already satisfies symmetry. But for $(7, 8)$ we need $(8, 7)$, and for $(11, 13)$ we need $(13, 11)$. These two ordered pairs are missing, so we must add them. Hence minimum elements to add = 2.

COMMON TRAPS

- **Listing x values incorrectly by not enforcing divisibility by 4.** Since $M = \{1, 2, 3, \dots, 16\}$, both coordinates of every ordered pair must be integers from 1 to 16. So $5x - 3$ must be divisible by 4. *Avoid it:* Use the modular check $5x - 3 \equiv x - 3 \pmod{4}$, so $x \equiv 3 \pmod{4}$, then test $x = 3, 7, 11, 15$ and reject $x = 15$ because $y = 18$ not in M .
- **Forgetting to check upper bound $y \leq 16$.** A pair belongs to R only if both x and y lie in M . Although $x = 15$ is in M , $y = 18$ is not. *Avoid it:* After each candidate x , run two checks: integer check and range check $1 \leq y \leq 16$.

REVISE The definitions of symmetric and reflexive relations side by side. · Use modular arithmetic to list integer solutions quickly: here $5x - 3$ divisible by 4 means $x \equiv 3 \pmod{4}$.

7 Answer: C

Ref 11405

Hard ~1.5 min Counting ordered pairs satisfying inequality

QUICK APPROACH Use the concept of counting lattice points inside an ellipse by first determining the integer values of x that satisfy the inequality and then counting the corresponding y values for each x .

Fix x , then count the integers y that fit.

Step 1. $4x^2 < 52$ gives $x^2 < 13$, so $x \in \{-3, -2, \dots, 3\}$.

Step 2. $x = 0$: $y^2 < 52$, so $|y| \leq 7$, giving 15 values.

Step 3. $x = \pm 1$: $y^2 < 48$, so $|y| \leq 6$, giving 13 values each, 26 in all.

Step 4. $x = \pm 2$: $y^2 < 36$, so $|y| \leq 5$. The value $y = \pm 6$ fails because the inequality is strict. That is 11 values each, 22 in all.

Step 5. $x = \pm 3$: $y^2 < 16$, so $|y| \leq 3$, giving 7 values each, 14 in all.

Step 6. Total = $15 + 26 + 22 + 14 = 77$.

Answer: (C) 77

8 Answer: A

Ref 11488

Hard ~2 min Properties of Relations (Reflexive and Symmetric)

Given $A = \{-2, -1, 0, 1, 2, 3, 4\}$. Relation R is defined by $2x + y \leq 2$.

Step 1: Count elements in R (I).For each $x \in A$, find $y \in A$ satisfying $2x + y \leq 2$. $x = -2 \rightarrow y \leq 6 \rightarrow$ all 7 elements $x = -1 \rightarrow y \leq 4 \rightarrow$ all 7 elements $x = 0 \rightarrow y \leq 2 \rightarrow \{-2, -1, 0, 1, 2\} \rightarrow 5$ $x = 1 \rightarrow y \leq 0 \rightarrow \{-2, -1, 0\} \rightarrow 3$ $x = 2 \rightarrow y \leq -2 \rightarrow \{-2\} \rightarrow 1$ $x = 3 \rightarrow y \leq -4 \rightarrow$ none $x = 4 \rightarrow y \leq -6 \rightarrow$ noneSo $l = 7 + 7 + 5 + 3 + 1 = 23$.

Step 2: Make relation reflexive.

Reflexive requires $(a, a) \in R$ for all $a \in A$.Condition: $2a + a \leq 2 \Rightarrow 3a \leq 2 \Rightarrow a \leq 2/3$.Elements satisfying: $-2, -1, 0$ (3 pairs already present).Total required = 7 pairs $\rightarrow$ missing = 4.Thus $m = 4$.

Step 3: Make relation symmetric.

If $(x, y) \in R$ then (y, x) must also be in R .

Missing mirror pairs are:

 $(3, -2), (4, -2), (2, -1), (3, -1), (4, -1), (2, 0)$.

Total additions = 6.

Thus $n = 6$.

Step 4: Final value

 $l + m + n = 23 + 4 + 6 = 33$.**9 Answer: D**

Ref 11588

Medium ~1 min Equivalence relations / Congruence modulo n

Partition A by residues mod 3: class 0 = $\{0, 3, 6, 9\}$ (4 elements), class 1 = $\{1, 4, 7\}$ (3 elements), class 2 = $\{2, 5, 8\}$ (3 elements).

Pairs in R are exactly ordered pairs where both elements lie in the same residue class. So $n(R) = 4^2 + 3^2 + 3^2 = 16 + 9 + 9 = 34$, not 36; Statement I is false. R is reflexive because $|x - x| = 0$ (multiple of 3) for all x ; symmetric because $|x - y|$ multiple of 3 implies $|y - x|$ is multiple of 3; transitive because if $x \equiv y \pmod{3}$ and $y \equiv z \pmod{3}$ then $x \equiv z \pmod{3}$. Hence R is an equivalence relation and Statement II is true. Therefore option 4 is correct.

10 Answer: B

Ref 11590

Medium ~1 min Number of onto functions (Surjections) / Counting

QUICK APPROACH Start by finding the elements of sets A and B by solving the given inequalities and equations, then use the formula for the number of onto (surjective) functions between finite sets.

Find A : $||x-3| - 3| \leq 1 \Rightarrow -1 \leq |x-3| - 3 \leq 1 \Rightarrow 2 \leq |x-3| \leq 4$. So $|x-3| = 2, 3, 4 \Rightarrow x = 1, 5$ (for 2); $x = 0, 6$ (for 3); $x = -1, 7$ (for 4). Hence $A = \{-1, 0, 1, 5, 6, 7\}$ (6 elements).

Find B : solve $(x-2)(x-4)/(x-1) \cdot \ln(|x-2|) = 0$. Product is zero when either $(x-4)=0$ or $\ln(|x-2|)=0$, provided expression is defined. $x=4$ is valid. $\ln(|x-2|)=0 \Rightarrow |x-2|=1 \Rightarrow x=3$ or $x=1$, but

$x=1$ makes denominator zero (undefined), and $x=2$ is not valid because $\ln 0$ undefined. Thus $B = \{3, 4\}$ (2 elements).

Number of onto (surjective) functions from a 6-element set to a 2-element set = total functions 2^6 minus the 2 constant (non-surjective) functions = $64 - 2 = 62$.

COMMON TRAPS

- **Dropping the lower bound after removing the outer absolute value in A.** $|u| \leq 1$ means $-1 \leq u \leq 1$. Here $u = |x-3| - 3$, so $-1 \leq |x-3| - 3 \leq 1$, giving $2 \leq |x-3| \leq 4$. *Avoid it:* Every time an absolute-value inequality is of the form $|u| \leq a$, immediately write the two-sided inequality $-a \leq u \leq a$.
- **Ignoring the integer restriction in set A.** $A = \{x \in \mathbb{Z} : \dots\}$ means only integer x are allowed. After obtaining intervals, take only integer points. *Avoid it:* Circle the domain condition in set-builder notation before solving.

REVISE Nested absolute-value inequalities with integer domains.

• Practice domain checking in equations involving rational expressions and logarithms.

11 Answer: C

Ref 11662

Hard ~1.5 min Relation on Cartesian product / Counting ordered pairs

We must count ordered pairs $((a, b), (c, d))$ with a, b, c, d in $\{1, 2, 3, 4\}$ satisfying $2a + 3b = 3c + 4d$. Compute frequency of each possible value on left and right and multiply.

LHS values $s = 2a + 3b$ (a, b in $1..4$). The multiset of s and frequencies is:

$5(1), 7(1), 8(1), 9(1), 10(1), 11(2), 12(1), 13(1), 14(2), 15(1), 16(1), 17(1), 18(1), 20(1)$ — total 16 pairs on LHS.

RHS values $t = 3c + 4d$ (c, d in $1..4$). Each of the 16 values $\{7, 10, 13, 16, 11, 14, 17, 20, 15, 18, 21, 24, 19, 22, 25, 28\}$ appears exactly once.

Intersection of possible values = $\{7, 10, 11, 13, 14, 15, 16, 17, 18, 20\}$.

For each common value v the number of relation pairs equals $\text{freq_LHS}(v) \cdot \text{freq_RHS}(v)$. Since $\text{freq_RHS}(v) = 1$ for all, total number = sum of freq_LHS over the intersection = $1+1+2+1+2+1+1+1+1+1 = 12$.

Thus the number of elements in R is 12.**12 Answer: C**

Ref 11753

Medium ~1 min Truth Table of Logic Circuit

From the circuit the output is 1 only when $A = 1$ and $B = 0$, so the Boolean function is $Y = A \cdot B'$. Constructing the truth table for $Y = A \cdot B'$ gives: $(A, B) = (0, 0) \rightarrow 0; (0, 1) \rightarrow 0; (1, 0) \rightarrow 1; (1, 1) \rightarrow 0$. Hence option C is correct.

COMMON TRAPS

- **Treating the circuit as XOR because option A looks familiar.** The explanation states the actual function is " $Y = A \cdot B'$ ", not XOR. AND with NOT- B is 1 only for $A=1, B=0$. *Avoid it:* Before comparing options, write the intermediate signal B' and compute $A \cdot B'$ row by row.
- **Forgetting the NOT on B in $Y = A \cdot B'$.** B' means complement: if $B=0, B'=1$; if $B=1, B'=0$. *Avoid it:* Add a separate B' column before computing Y .

REVISE Always create intermediate columns such as B' before calculating Y . • Translate Boolean symbols into words: dot = AND, apostrophe = NOT.

13 Answer: C

Ref 11871

Hard ~1 min Injective (one-one) and Many-one functions

Statement I: $f(x) = x/(1+|x|)$. For $x \geq 0$, $|x| = x$ so $f(x) = x/(1+x)$, whose derivative is $1/(1+x)^2 > 0$, so f is strictly increasing on $[0, \infty)$ with range $[0, 1)$. For $x < 0$, $|x| = -x$ so $f(x) = x/(1-x)$, derivative $1/(1-x)^2 > 0$, so f is strictly increasing on $(-\infty, 0)$ with range $(-1, 0)$. The two ranges are disjoint except the common value 0 at $x=0$, so f is injective (one-one).

Statement II: $f(x) = (x^2 + 4x - 30)/(x^2 - 8x + 18)$. To show it's many-one, find distinct x giving same value. Compute $f(0) = (-30)/(18) = -5/3$. Solve $(x^2 + 4x - 30)/(x^2 - 8x + 18) = -5/3$: $3(x^2 + 4x - 30) = -5(x^2 - 8x + 18) \Rightarrow 8x^2 - 28x = 0 \Rightarrow 4x(2x - 7) = 0$ so $x=0$ or $x=7/2$. Thus $f(0) = f(7/2) = -5/3$ with $0 \neq 7/2$, so f is many-one (not injective). Therefore both Statement I and Statement II are true.

COMMON TRAPS

- **Concluding one-one from increasing pieces alone.**

$\frac{x}{1+|x|}$ is increasing on each side of 0 and the two sides have disjoint ranges, so it is one-one on $\mathbb{R}$. *Avoid it:* Check that the ranges of the pieces do not overlap.

- **Assuming a rational function must be one-one.** A rational function of degree 2 over degree 2 usually takes some value twice. *Avoid it:* Set $f(x) = f(0)$ and look for a second root.

REVISE One-one and many-one tests for functions.

14 Answer: 2

Ref 10665

Medium ~3 min Sets and their representation

Each fraction in B is already in lowest terms, so count the coprime pairs.

Step 1. Since $\gcd(m, n) = 1$, different pairs (m, n) give different fractions. So $n(B)$ is the number of pairs with $1 \leq m < n \leq 10$ and $\gcd(m, n) = 1$.

Step 2. For a fixed n , the number of such m is Euler's totient $\varphi(n)$.

Step 3. The values $\varphi(2), \varphi(3), \dots, \varphi(10)$ are 1, 2, 2, 4, 2, 6, 4, 6, 4.

Step 4. Their sum is $1 + 2 + 2 + 4 + 2 + 6 + 4 + 6 + 4 = 31$.

Answer: (2) 31

COMMON TRAPS

- **Counting all ordered pairs with $m < n$ without applying coprime condition.** B contains only fractions m/n with m, n in A , $m < n$, and $\gcd(m, n) = 1$ simultaneously. *Avoid it:* For each denominator n , list only numerators $m < n$ that are coprime to n ; count by $\varphi(n)$.
- **Confusing number of fractions with number of possible m and n values.** $n(B)$ means cardinality of set B , not the denominator variable n . *Avoid it:* Rewrite $n(B)$ as $|B|$ before solving.

REVISE Cardinality notation $n(S)$ or $|S|$. Practice Euler totient values $\varphi(1)$ to $\varphi(10)$.

15 Answer: 3

Ref 10757

Medium ~6 min Range of Function

QUICK APPROACH Use the derivative to find the least and greatest values of f on $[0, 3]$, and write $g(x)$ as 1 minus a fraction.

Find the ranges A and B of the two onto functions, then count the integers in their union.

Step 1. $f'(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3)$. On $[0, 3]$, f increases on $[0, 2]$ and decreases on $[2, 3]$.

Step 2. $f(0) = 7$, $f(2) = 16 - 60 + 72 + 7 = 35$ and $f(3) = 54 - 135 + 108 + 7 = 34$. As f is continuous and onto, $A = [7, 35]$, which holds the 29 integers 7, 8, ..., 35.

Step 3. For $x \geq 0$, $g(x) = 1 - \frac{1}{x^{2025} + 1}$. It is 0 at $x = 0$, increases, and stays below 1 for every x , so $B = [0, 1)$. The only integer in B is 0.

Step 4. The integers of A and B do not overlap, so $n(S) = 29 + 1 = 30$.

Answer: (3) 30

COMMON TRAPS

- **Taking the endpoint value as the maximum.** The cubic has a local maximum at $x = 2$, where $f(2) = 35$ is larger than $f(3) = 34$. *Avoid it:* Compare the values at every critical point inside the interval as well as at both endpoints.
- **Counting 1 as an element of B .** $g(x)$ approaches 1 as x grows but never equals it, so 1 is not in B . *Avoid it:* Check whether a limiting value is actually attained before putting it in a range.

REVISE Finding the range of a function using derivatives. · Onto functions and integer counting in intervals.

16 Answer: 2

Ref 10784

Medium ~4 min Relations

Step 1: To determine if R is an equivalence relation, we check if it is reflexive, symmetric, and transitive.

• **Reflexive:** For $(a_1, b_1) \in X$, $(a_1, b_1)R(a_1, b_1)$ as $b_1 = b_1$. Thus, R is reflexive.

• **Symmetric:** If $(a_1, b_1)R(a_2, b_2)$, then $b_1 = b_2$. It follows that $(a_2, b_2)R(a_1, b_1)$ as $b_2 = b_1$. Thus, R is symmetric.

• **Transitive:** If $(a_1, b_1)R(a_2, b_2)$ and $(a_2, b_2)R(a_3, b_3)$, then $b_1 = b_2$ and $b_2 = b_3$. Therefore, $b_1 = b_3$, $(a_1, b_1)R(a_3, b_3)$. Thus, R is transitive.

Since R is reflexive, symmetric, and transitive, Statement I is true.

Step 2: The set $S = \{(x, y) \in X : (x, y)R(a, b)\} = \{(x, y) \in X : y = b\}$. This represents a line parallel to the x -axis (horizontal line at $y = b$), not $y = x$. Hence, Statement II is false.

Thus, the correct answer is option (2): Statement I is true but Statement II is false.

COMMON TRAPS

- **Testing reflexivity using two different ordered pairs instead of the same pair.** For reflexivity on X , check " $(a_1, b_1)R(a_1, b_1)$ ", which uses $b_1 = b_1$, always true. *Avoid it:* Replace the second element by the first element before applying the relation condition.
- **Thinking equality of ordered pairs is required in the relation.** The variable a_1 and a_2 are irrelevant for R ; only b_1 and b_2 matter. *Avoid it:* Underline the condition after $\Leftrightarrow$: only " $b_1 = b_2$ " is to be tested.

REVISE When a relation is defined on ordered pairs, explicitly map each symbol in the test expression to the variables in the definition. · For equivalence relation questions, check reflexive, symmetric, and transitive mechanically before interpreting geometry.

17 Answer: A

Ref 11501

Medium ~1 min Properties of relations (reflexive, symmetric, transitive)

QUICK APPROACH Start by checking the definitions of reflexive, symmetric, and transitive properties for relations, and test these on the given condition $f(o)=g(1)$ and $f(1)=g(o)$.

Check properties: 1) Symmetric: If $(f, g) \in R$ then $f(o)=g(1)$ and $f(1)=g(o)$. Swapping f and g gives $g(o)=f(1)$ and $g(1)=f(o)$, so $(g, f) \in R$. Thus R is symmetric. 2) Reflexive: For (f, f) to be in R we need $f(o)=f(1)$. Not every function satisfies this (pick f with $f(o) \neq f(1)$), so R is not reflexive. 3) Transitive: Find counterexample. Let f, g, h satisfy $(f, g) \in R$ and $(g, h) \in R$ but $(f, h) \notin R$. Take values $f(o)=0, f(1)=1; g(o)=1, g(1)=0; h(o)=0, h(1)=1$. Then $f(o)=g(1)=0$ and $f(1)=g(o)=1$ so $(f, g) \in R$; $g(o)=h(1)=1$ and $g(1)=h(o)=0$ so $(g, h) \in R$. But $f(o)=0 \neq h(1)=1$, so $(f, h) \notin R$. Hence R is not transitive. Therefore R is symmetric but neither reflexive nor transitive.

COMMON TRAPS

- **Treating reflexivity as automatic because the ordered pair is (f, f) .** For reflexivity, every f in A must satisfy $f(o)=f(1)$, which is not true for all integer-valued functions. *Avoid it:* Before deciding reflexive, explicitly write the condition for (f, f) : $f(o)=f(1)$ and $f(1)=f(o)$.
- **Chaining the wrong equalities while testing transitivity.** The correct chaining gives $f(o)=g(1)=h(o)$ and $f(1)=g(o)=h(1)$, which is not the required $f(o)=h(1)$ and $f(1)=h(o)$. *Avoid it:* Write the two assumptions and the required conclusion in three separate lines before chaining.

REVISE For relation properties on function sets, always rewrite the defining condition for (f, f) , (g, f) , and (f, h) explicitly. · Use counterexamples with a small value table for $f(o), f(1), g(o), g(1), h(o), h(1)$.

18 Answer: A

Ref 11988

Hard ~2 min Roots of quadratic (conditions for positive roots)

Let $A = 1-a, B = 2(a-3), C = 9$. For a quadratic $Ax^2 + Bx + C = 0$ to have both roots positive (with $A \neq 0$) we need: (i) $\Delta = B^2 - 4AC \geq 0$, (ii) product $r_1 r_2 = C/A > 0$, (iii) sum $r_1 + r_2 = -B/A > 0$. Compute $\Delta: \Delta = [2(a-3)]^2 - 4(1-a)(9) = 4[(a-3)^2 - 9(1-a)] = 4[a^2 - 6a + 9 - 9 + 9a] = 4[a(a+3)]$. So $\Delta \geq 0 \Rightarrow a(a+3) \geq 0 \Rightarrow a \in (-\infty, -3] \cup [0, \infty)$. Product condition: $9/(1-a) > 0 \Rightarrow 1-a > 0 \Rightarrow a < 1$. Combining with Δ gives $a \in (-\infty, -3] \cup [0, 1)$. Sum condition: $-B/A = -2(a-3)/(1-a) > 0$. Simplifying gives $2(a-3)/(a-1) > 0$ which holds for $a \in (-\infty, 1)$ and $(3, \infty)$; intersecting with $a < 1$ yields $a \in (-\infty, 1)$. Together all three conditions restrict a to $(-\infty, -3] \cup [0, 1)$. Thus this set = $(-\infty, -\alpha] \cup [\beta, \gamma)$ with $-\alpha = -3 \Rightarrow \alpha = 3, \beta = 0, \gamma = 1$. Therefore $2\alpha + \beta + \gamma = 2 \cdot 3 + 0 + 1 = 7$.

COMMON TRAPS

- **Treating the excluded value $a=1$ as allowed/linear case.** The roots of a quadratic require $A=1-a \neq 0$; $a=1$ is excluded and the interval must be open at 1. *Avoid it:* Before applying quadratic root conditions, write $A=1-a \neq 0$ and mark $a=1$ on the number line as a forbidden point.
- **Wrong sign in sum of roots condition.** For $Ax^2 + Bx + C = 0, r_1 + r_2 = -B/A$, not B/A . *Avoid it:* Write Vieta's formulas explicitly before substituting A, B, C .

REVISE Vieta's conditions for both roots positive: $\Delta \geq 0, C/A > 0, -B/A > 0$ with $A \neq 0$. · Practice rational inequality sign charts with forbidden points marked explicitly.

19 Answer: D

Ref 12088

Hard ~2 min Reflexive and Symmetric properties of relations

Compute R : for each x in $A, y = \max\{x, 1\}$. So

- $x = -2 \rightarrow y = 1$ gives $(-2, 1)$
- $x = -1 \rightarrow y = 1$ gives $(-1, 1)$
- $x = 0 \rightarrow y = 1$ gives $(0, 1)$
- $x = 1 \rightarrow y = 1$ gives $(1, 1)$
- $x = 2 \rightarrow y = 2$ gives $(2, 2)$
- $x = 3 \rightarrow y = 3$ gives $(3, 3)$

Thus $R = \{(-2, 1), (-1, 1), (0, 1), (1, 1), (2, 2), (3, 3)\}$ so $l = 6$.

To make R reflexive we need (a, a) for every a in A . Present are $(1, 1), (2, 2), (3, 3)$. Missing are $(-2, -2), (-1, -1), (0, 0)$ so minimum $m = 3$.

To make R symmetric we need for every (a, b) in R the pair (b, a) . Pairs with $a \neq b$ are $(-2, 1), (-1, 1), (0, 1)$; their symmetric partners $(1, -2), (1, -1), (1, 0)$ are missing. Adding these three makes R symmetric; the diagonal pairs are already symmetric. So minimum $n = 3$.

Therefore $l + m + n = 6 + 3 + 3 = 12$.

COMMON TRAPS

- **Treating $\max\{x, 1\}$ incorrectly for negative x .** For $x = -2, -1, 0, \max\{x, 1\} = 1$; for $x = 1, 2, 3, \max\{x, 1\} = x$. *Avoid it:* Make a two-column table with x and $\max\{x, 1\}$ for all six elements before counting anything.
- **Counting outputs instead of ordered pairs for l .** Elements of R are ordered pairs, not single outputs. Here R has one ordered pair for each x in A , so $l = 6$. *Avoid it:* Write R explicitly as a set of ordered pairs before finding l .

REVISE Always expand a relation on a finite set into ordered pairs before testing reflexive/symmetric properties. · Use separate checklists: all diagonal pairs for reflexivity; all non-diagonal reverse pairs for symmetry.

20 Answer: C

Ref 12149

Hard ~1.5 min Relations (Reflexive)

Compute all ordered pairs (x, y) in $A \times A$ satisfying $0 \leq x^2 + 2y \leq 4$. For each x in A find allowed y .

- For $x = 0: x^2 = 0$ so $0 \leq 2y \leq 4 \Rightarrow 0 \leq y \leq 2$ gives $y = \{0, 1, 2\}$ (3 pairs).
- For $x = \pm 1: x^2 = 1$ so $0 \leq 1 + 2y \leq 4 \Rightarrow -0.5 \leq y \leq 1.5$ gives $y = \{0, 1\}$ (2 each, total 4).
- For $x = \pm 2: x^2 = 4$ so $0 \leq 4 + 2y \leq 4 \Rightarrow -2 \leq y \leq 0$ gives $y = \{-2, -1, 0\}$ (3 each, total 6).
- For $x = \pm 3: x^2 = 9$ so $0 \leq 9 + 2y \leq 4 \Rightarrow -4.5 \leq y \leq -2.5$ gives $y = \{-3\}$ (1 each, total 2).

Adding counts: $3 + 4 + 6 + 2 = 15$, so $l = 15$.

Next check reflexive pairs (a, a) : need $0 \leq a^2 + 2a \leq 4$.

Compute $a^2 + 2a$: for $a = -3: 9-6=3$ (ok), $-2: 4-4=0$ (ok), $-1: 1-2=-1$ (no), $0: 0$ (ok), $1: 1+2=3$ (ok), $2: 4+4=8$ (no), $3: 9+6=15$ (no). Thus diagonal pairs present are for $a = -3, -2, 0, 1$ (4

present). Missing diagonals are for $a = -1, 2, 3$ (3 missing), so $m = 3$.

Therefore $l + m = 15 + 3 = 18$.

COMMON TRAPS

- **Counting the number of valid x-values instead of ordered pairs (x,y).** Here l is the number of elements of R , i.e. the number of ordered pairs (x,y) , not the number of x -values. For each $x \in A$, count all $y \in A$ satisfying the inequality and add those counts. *Avoid it:* Make a two-column table: x , allowed y . The count contribution of each row is $|\text{allowed } y|$.
- **Forgetting that y must belong to A.** A relation on A means both x and y must be elements of A . Even if an inequality interval contains other integers/reals, only values in $\{-3, -2, -1, 0, 1, 2, 3\}$ are allowed. *Avoid it:* After each interval for y , intersect it explicitly with A : allowed $y = \text{interval} \cap A$.

REVISE The definition of a relation on A as a subset of $A \times A$, so both coordinates must lie in A . Practice solving compound inequalities and then intersecting the result with a finite set.

21 Answer: D

Ref 12119

Medium ~1.5 min Counting, Set Intersection, Injective Functions

Step 1: Count elements of C . $C = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$.

Integer solutions: $x=0 \rightarrow y = -2, -1, 0, 1, 2$ (5 points). $x=\pm 1 \rightarrow y = -1, 0, 1$ (3 each $\rightarrow 6$). $x=\pm 2 \rightarrow y=0$ (1 each $\rightarrow 2$). Total $n(C) = 5+6+2 = 13$.

Step 2: Find $D = A \cap B$, intersection of circles $x^2 + y^2 = 25$ and $x^2 + 9y^2 = 144$. Subtract the first from the second: $(x^2 + 9y^2) - (x^2 + y^2) = 144 - 25 \Rightarrow 8y^2 = 119 \Rightarrow y^2 = 119/8 > 0$, so $y = \pm\sqrt{119/8}$.

Then $x^2 = 25 - y^2 = 25 - 119/8 = 81/8 \Rightarrow x = \pm\sqrt{81/8} = \pm 9/\sqrt{8}$. Both signs of x and y are independent, so there are 4 distinct real intersection points. Thus $n(D) = 4$.

Step 3: Number of one-to-one (injective) functions from D (size 4) to C (size 13) equals number of ordered injections = permutations $P(13, 4) = 13 \times 12 \times 11 \times 10 = 17160$.

Therefore the correct answer is 17160.

COMMON TRAPS

- **Treating C as a real disk instead of integer lattice points.** For C , both x and y must be integers; list lattice points only. *Avoid it:* Circle or underline the domain symbol before counting: $R \times R$ means continuum; $Z \times Z$ means integer pairs.
- **Counting only boundary points of C.** The symbol $\leq$ includes the boundary and all interior lattice points. *Avoid it:* For each candidate x , solve $y^2 \leq 4 - x^2$, not $y^2 = 4 - x^2$.

REVISE Before solving, annotate each set with its universe: $A, B \subset R \times R$; $C \subset Z \times Z$; $D = A \cap B \subset R \times R$. For equations containing only squares, explicitly count sign choices after finding x^2 and y^2 .

22 Answer: C

Ref 12281

Hard ~2 min Relations (reflexive, symmetric) and counting ordered pairs

We need all ordered pairs (x,y) in $A \times A$ with $2x - y \in \{0, 1\}$.

1) $2x - y = 0 \Rightarrow y = 2x$. For $x \in A$, $2x \in A$ only for $x = -1, 0, 1$ giving pairs: $(-1, -2), (0, 0), (1, 2)$. (3 pairs)

2) $2x - y = 1 \Rightarrow y = 2x - 1$. For $x \in A$, $2x - 1 \in A$ for $x = -1, 0, 1, 2$ giving pairs: $(-1, -3), (0, -1), (1, 1), (2, 3)$. (4 pairs)

So $l = 3 + 4 = 7$.

To be reflexive we need all (a,a) for 7 elements of A . Currently only $(0,0)$ and $(1,1)$ are present (since $2x - x = x$ must be 0 or 1). So we must add the other 5 diagonal pairs. Thus $m = 5$.

For symmetry: check off-diagonal pairs in R : $(-1, -2), (1, 2), (-1, -3), (0, -1), (2, 3)$. In each case the reverse (y,x) is not in R , so

we must add each reverse—5 additions. Thus $n = 5$.

Therefore $l + m + n = 7 + 5 + 5 = 17$.

COMMON TRAPS

- **Counting diagonal pairs using only $x=0$, missing $(1,1)$.** For reflexive pairs (a,a) , substitute $x=y=a$: $2a - a = a$, so a must be in $\{0, 1\}$. Both $(0,0)$ and $(1,1)$ already exist. *Avoid it:* When a condition has a set like $\{0, 1\}$, test both allowed values before counting missing diagonal pairs.
- **Omitting negative x values while solving $y=2x-1$.** $A = \{-3, -2, -1, 0, 1, 2, 3\}$; every x in A must be tested, including $x=-1$, for which $y=-3 \in A$. *Avoid it:* Make a two-column table for all seven x -values in A and check whether y belongs to A .

REVISE Always enumerate $x \in A$ completely before counting l . For reflexivity, substitute $y=x$ and check every diagonal pair (a,a) .

23 Answer: A

Ref 12294

Hard ~2 min Arithmetic Progression (Product of terms of an AP)

Let the terms of A be $a - d, a, a + d$ and of B be $a - D, a, a + D$. Given sum of elements in each set is 36, so $3a = 36 \Rightarrow a = 12$. The product of elements of A and B are $p = a(a^2 - d^2)$ and $q = a(a^2 - D^2)$. Given $D = d + 3$. With $a = 12$ we have

$$p = 12(144 - d^2) = 1728 - 12d^2$$

$$q = 12(144 - (d + 3)^2) = 12(144 - d^2 - 6d - 9) = 1620 - 12d^2 - 72d.$$

Compute $p + q$ and $p - q$:

$$p + q = (1728 - 12d^2) + (1620 - 12d^2 - 72d) = 3348 - 24d^2 - 72d$$

$$p - q = (1728 - 12d^2) - (1620 - 12d^2 - 72d) = 108 + 72d.$$

Given $(p + q)/(p - q) = 19/5$. Substitute and solve:

$$\frac{3348 - 24d^2 - 72d}{108 + 72d} = \frac{19}{5}.$$

Cross-multiplying: $5(3348 - 24d^2 - 72d) = 19(108 + 72d)$.

This simplifies to $-120d^2 - 1728d + 14688 = 0$. Divide by -24 to get

$$5d^2 + 72d - 612 = 0. \text{ Discriminant } \Delta = 72^2 + 4 \cdot 5 \cdot 612 = 17424 = (132)^2.$$

So $d = \frac{-72 \pm 132}{10}$, giving $d = 6$ (positive root).

Then $D = d + 3 = 9$. Now compute p and q :

$$p = 12(144 - 36) = 12 \cdot 108 = 1296$$

$$q = 12(144 - 81) = 12 \cdot 63 = 756$$

$$\text{Hence } p - q = 1296 - 756 = 540.$$

24 Answer: 4

Ref 9117

Medium ~2 min Symmetric Relations

QUICK APPROACH Start by checking which pairs (x, y) satisfy the relation and then verify whether their reversed pairs (y, x) are also present to determine symmetry.

For R_1 : Solve $2x - 3y = 2$ with $x, y \in \{1, \dots, 20\}$. This yields $y = \frac{2(x-1)}{3}$, so $x = 3k + 1, y = 2k$ for integer $k \geq 1$. With

bounds, $k = 1$ to 6 gives exactly the pairs $(4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)$. None satisfy $x = y$ or have their reverses already in R_1 (e.g., $2 \cdot 2 - 3 \cdot 4 = -8 \neq 2$). The 6 reverses are distinct, so the symmetric closure requires adding precisely these 6; thus $M = 6$.

Option [3] (8) encodes the specific mistake of undercounting pairs in R_1 as 4 (perhaps conflating with R_2 's count), yielding erroneous $4 + 4 = 8$.

For $R_2: -5x + 4y = 0$ gives $y = \frac{5x}{4}$, so $x = 4k, y = 5k$ for $k = 1$ to 4 : $(4, 5), (8, 10), (12, 15), (16, 20)$. Again, no self-pairs and 4 distinct reverses needed, so $N = 4$.

Hence $M + N = 10$. The correct option is [4].

COMMON TRAPS

- **Solving R_2 as $y = \frac{4x}{5}$.** $-5x + 4y = 0$ gives $y = \frac{5x}{4}$, so x must be a multiple of 4. *Avoid it:* Rearrange each equation carefully and list the pairs that stay inside X .
- **Counting unordered pairs instead of missing reverses.** Each pair (x, y) whose reverse (y, x) is not already present needs one new element. *Avoid it:* For every listed pair, check whether its reverse already satisfies the relation.

REVISE Symmetric closure of a relation.

25 Answer: 46

Ref 9132

Medium ~4 min Relations

QUICK APPROACH Start by expressing the number of pairs in R_1 and R_2 using the definition of divisibility and then find their intersection based on mutual divisibility conditions.

Count the ordered pairs with $a \mid b$, then remove those that also have $b \mid a$.

Step 1. R_1 holds the pairs (a, b) with $a \mid b$. For a fixed a , the number of such b in A is $\lfloor \frac{20}{a} \rfloor$.

Step 2. Sum over $a = 1$ to 20 : $20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2 + 2$ for $a = 1$ to 10 , plus 1 for each of $a = 11$ to 20 . This gives $56 + 10 = 66$.

Step 3. R_2 holds the pairs with $b \mid a$. A pair lies in both relations when $a \mid b$ and $b \mid a$, which for positive integers means $a = b$. There are 20 such pairs.

Step 4. $R_1 - R_2$ removes from R_1 only the pairs that are also in R_2 : $66 - 20 = 46$.

Answer: 46

COMMON TRAPS

- **Subtracting all of R_2 instead of the common part.** $R_1 - R_2$ removes only the pairs that lie in both relations. *Avoid it:* Find $R_1 \cap R_2$ first, then subtract its size from $|R_1|$.
- **Using $\frac{20}{a}$ instead of its floor.** The number of multiples of a up to 20 is $\lfloor \frac{20}{a} \rfloor$. *Avoid it:* Round each count down to a whole number before adding.

REVISE Set difference with ordered pairs.

26 Answer: 4

Ref 9181

Medium ~2 min Relations

To verify statement (I), first check reflexivity of R on $\mathbb{N} \times \mathbb{N}$. For any (a, b) , $(a, b)R(a, b)$ holds because $a \leq a$ (or $b \leq b$) is true. Thus, R is reflexive.

For symmetry, consider $(1, 1)R(2, 2)$: $1 \leq 2$ is true, so it holds. But $(2, 2)R(1, 1)$ requires $2 \leq 1$ or $2 \leq 1$, both false. Hence, R is not symmetric. Statement (I) is correct.

For (II), check transitivity. Option 1 encoded the common mistake of assuming transitivity without counterexample verification. Consider $(3, 3)R(4, 1)$: $3 \leq 4$ holds. Also,

$(4, 1)R(2, 2)$ holds as $1 \leq 2$. But $(3, 3)R(2, 2)$ fails: neither $3 \leq 2$ nor $3 \leq 2$ is true. Thus, R is not transitive, so (II) is false.

Only (I) is correct, i.e., option [4].

COMMON TRAPS

- **Reading or as and in the definition.** The relation needs only one of $x_1 \leq x_2$ or $y_1 \leq y_2$ to hold. *Avoid it:* Test transitivity with a pair that satisfies one inequality and fails the other.
- **Assuming transitivity holds because each inequality is transitive.** $(3, 3)R(1, 4)$ and $(1, 4)R(2, 1)$ hold, but $(3, 3)$ is not related to $(2, 1)$, since $3 > 2$ and $3 > 1$. *Avoid it:* Look for a counterexample where a different coordinate works in each of the two steps.

REVISE Reflexive, symmetric and transitive tests.

27 Answer: 2

Ref 9282

Medium ~4 min Relations

The relation on $A \times B$ is defined by $(a, b)R(c, d)$ iff $3ad - 7bc$ is even. Modulo 2, this simplifies to $ad \equiv bc \pmod{2}$ since $3 \equiv 1$ and $-7 \equiv 1 \pmod{2}$. Equivalently, ad and bc have the same parity: both even or both odd.

Reflexive: For $(a, b)R(a, b)$, $ab \equiv ba \pmod{2}$ holds identically, so R is reflexive.

Symmetric: If $ad \equiv bc \pmod{2}$, then for $(c, d)R(a, b)$ we need $cb \equiv da \pmod{2}$, which is identical. Thus R is symmetric. (Option 4 claimed it is not symmetric, likely from failing to reduce coefficients modulo 2 and incorrectly comparing $3ad - 7bc$ to $3cb - 7da$.)

Not transitive: Consider pairs $P = (3, 4)$, $Q = (2, 4)$, $S = (3, 1)$. Then $3 \cdot 4 \equiv 4 \cdot 2 \pmod{2}$ (both even), so $P R Q$. Also $2 \cdot 1 \equiv 4 \cdot 3 \pmod{2}$ (both even), so $Q R S$. But $3 \cdot 1 \equiv 4 \cdot 3 \pmod{2}$ fails (odd $\not\equiv$ even), so not $P R S$. Hence not transitive.

R is reflexive and symmetric but not transitive, i.e., option [2].

COMMON TRAPS

- **Treating R as a relation on A or B instead of on $A \times B$.** The elements related by R are ordered pairs from $A \times B$; both (a, b) and (c, d) are elements of $A \times B$, so $a, c \in A$ and $b, d \in B$. *Avoid it:* Before checking reflexive/symmetric/transitive, write the type: $R \subseteq (A \times B) \times (A \times B)$.
- **Missing the parity simplification modulo 2.** Modulo 2, $3 \equiv 1$ and $-7 \equiv 1$, so $3ad - 7bc$ is even iff $ad - bc$ is even, i.e., ad and bc have the same parity. *Avoid it:* Replace all odd coefficients by 1 modulo 2 before testing even/odd.

REVISE Relation properties when the base set itself is a Cartesian product. · Practice modulo-2 parity reductions of algebraic expressions.

28 Answer: 10

Ref 9295

Medium ~3 min Sets and their representation

QUICK APPROACH Write the union of the three sets by inclusion-exclusion, then use the fact that each pairwise intersection is at least as large as the triple intersection.

Use inclusion-exclusion on the three sets and bound each pairwise intersection from below by the triple intersection.

Step 1. Let $x = n(M \cap P)$, $y = n(P \cap C)$, $z = n(M \cap C)$ and

$t = n(M \cap P \cap C)$. Every student passed in at least one subject, so $40 = 20 + 25 + 16 - (x + y + z) + t$, which gives $x + y + z = 21 + t$.

Step 2. Each pairwise intersection contains the triple intersection, so $x \geq t, y \geq t, z \geq t$ and $x + y + z \geq 3t$. Hence $21 + t \geq 3t$, so $t \leq 10.5$ and $t \leq 10$.

Step 3. Check that $t = 10$ can happen. Take $x = 10, y = 10, z = 11$, which satisfy $x \leq 11, y \leq 15, z \leq 15$. Then Mathematics only is $20 - 10 - 11 + 10 = 9$, Physics only is $25 - 10 - 10 + 10 = 15$ and Chemistry only is $16 - 10 - 11 + 10 = 5$, all non-negative.

Step 4. The regions add to $9 + 15 + 5 + 0 + 0 + 1 + 10 = 40$, so the distribution is valid and the maximum is 10.

Answer: 10

COMMON TRAPS

- **Treating the at-most limits as exact counts.** The numbers 11, 15 and 15 are only upper bounds on the pairwise intersections, not their values. *Avoid it:* Keep the pairwise counts as unknowns and use the limits only to test a candidate distribution.
- **Not checking that the bound is reached.** An inequality gives only an upper limit, which is the answer only when a valid distribution attains it. *Avoid it:* Build one full Venn distribution with non-negative regions before accepting the bound.

REVIEW Inclusion-exclusion for three sets. · Maximum and minimum problems on Venn diagrams.

29 Answer: 960

Ref 9350

Medium ~3 min Relations

Count the symmetric relations, then remove those that are also reflexive.

Step 1. $A \times A$ has 16 ordered pairs: 4 diagonal pairs and 12 off-diagonal pairs.

Step 2. In a symmetric relation, (a, b) and (b, a) are in or out together. The 12 off-diagonal pairs form $\binom{4}{2} = 6$ such couples, giving 2^6 choices.

Step 3. Each diagonal pair can be in or out on its own, giving 2^4 choices. So there are $2^{10} = 1024$ symmetric relations.

Step 4. A symmetric relation that is also reflexive contains all 4 diagonal pairs, which leaves $2^6 = 64$ such relations.

Step 5. Symmetric but not reflexive: $1024 - 64 = 960$.

Answer: 960

COMMON TRAPS

- **Treating off-diagonal pairs as independent.** In a symmetric relation (a, b) and (b, a) must be chosen together. *Avoid it:* Count the 6 unordered off-diagonal couples, not the 12 ordered pairs.
- **Confusing not reflexive with irreflexive.** Not reflexive means at least one diagonal pair is missing, not that all are missing. *Avoid it:* Subtract the reflexive symmetric relations from all symmetric relations.

REVIEW Counting symmetric and reflexive relations.

30 Answer: 66

Ref 9712

Medium ~8 min

The smallest symmetric relation containing R is R together with all its reversed pairs.

Step 1. The condition $2x \equiv 3y$ here means $2x = 3y$. Then $3 \mid x$ and $2 \mid y$, so $x = 3k$ and $y = 2k$.

Step 2. We need $3k \leq 100$, so $k = 1, 2, \dots, 33$. Thus R has 33 pairs, from $(3, 2)$ to $(99, 66)$.

Step 3. For R_1 to be symmetric, every reverse $(2k, 3k)$ must be in R_1 . None of them is already in R , because $2(2k) = 3(3k)$ would need $k = 0$.

Step 4. No pair of R lies on the diagonal, since $2x = 3x$ forces $x = 0$. So all 33 reverses are new pairs.

Step 5. The minimum size is $n = 33 + 33 = 66$.

Answer: 66

COMMON TRAPS

- **Assuming a symmetric relation needs all diagonal pairs.** Symmetry needs only the reverse of each pair, not (x, x) for every x . *Avoid it:* Add only the reverses of pairs already present.
- **Answering with the size of R alone.** R_1 must contain R and every reverse pair, so its size is $|R \cup R^{-1}|$. *Avoid it:* Count the pairs of R , then add the reverses that are not already in R .

REVIEW Symmetric closure of a relation.

31 Answer: D

Ref 9721

Medium ~3 min Functions

To check if $f(x) = \frac{x^2 - 2x + 5}{x^2 - 4x + 9}$ ($\mathbb{R} \rightarrow \mathbb{R}$) is one-one, compute $f'(x)$. Let $u = x^2 - 2x + 5, v = x^2 - 4x + 9$. Then $u' = 2x - 2, v' = 2x - 4$, and numerator of f' is $2[(x - 1)v - u(x - 2)]$.

Expanding yields $2(-x^2 + 4x + 1)$. Since denominator $v^2 > 0$ always, $f'(x)$ has sign of $-x^2 + 4x + 1$, positive between roots $2 \pm \sqrt{5}$. Thus f increases on $(2 - \sqrt{5}, 2 + \sqrt{5})$ and decreases outside, so not monotonic (hence not one-one). Option A encodes the common mistake of assuming f is injective.

For onto, set $f(x) = y: (y - 1)x^2 + (2 - 4y)x + (9y - 5) = 0$. Discriminant $D = (2 - 4y)^2 - 4(y - 1)(9y - 5) = -20y^2 + 40y - 16 = -4(5y^2 - 10y + 4)$. Quadratic $5y^2 - 10y + 4 = 0$ has roots $1 \pm \frac{\sqrt{5}}{5}$; $D \geq 0$ only for y in $[1 - \frac{\sqrt{5}}{5}, 1 + \frac{\sqrt{5}}{5}]$, not all $\mathbb{R}$. Hence not onto.

The function is neither one-one nor onto, option [D].

COMMON TRAPS

- **Assuming a ratio of quadratics is one-one.** Here $f(0) = \frac{5}{9}$ and $f(-\frac{1}{2}) = \frac{5}{9}$, so two inputs give the same output. *Avoid it:* Solve $f(x) = f(a)$ for a simple a ; a second real solution shows the function is not one-one.
- **Assuming the function covers all reals.** The denominator never vanishes and both quadratics have the same degree, so the range is a bounded interval, not $\mathbb{R}$. *Avoid it:* Find the range by setting $y = f(x)$ and requiring a real x (discriminant ≥ 0).

REVISE One-one and onto functions · Range of rational functions

32 Answer: 1

Ref 9895

Medium ~3 min Relations

Build the smallest relation that is reflexive, symmetric and transitive and contains the given pairs.

Step 1. Reflexivity needs $(1, 1), (2, 2), (3, 3), (4, 4)$.

Step 2. Symmetry needs $(2, 1)$ and $(3, 1)$ along with $(1, 2)$ and $(1, 3)$.

Step 3. Transitivity: $(2, 1)$ and $(1, 3)$ give $(2, 3)$; $(3, 1)$ and $(1, 2)$ give $(3, 2)$.

Step 4. So 1, 2 and 3 are all related, and the classes are $\{1, 2, 3\}$ and $\{4\}$.

Step 5. The class $\{1, 2, 3\}$ gives $3 \times 3 = 9$ ordered pairs and $\{4\}$ gives 1 pair. Nothing forces 4 into the first class, so $|R| = 9 + 1 = 10$.

Answer: (1) 10

33 Answer: 3

Ref 9940

Medium ~3 min Sets and their representation

Step 1: Calculate the total number of integers in the range $[100, 700]$. There are $700 - 100 + 1 = 601$ integers.

Step 2: Calculate the number of multiples of 3 in this range. The smallest multiple of 3 in the range is 102 (3×34) and the largest is 699 (3×233), so there are $233 - 34 + 1 = 200$ multiples of 3.

Step 3: Calculate the number of multiples of 4. The smallest multiple of 4 is 100 (4×25) and the largest is 700 (4×175), so there are $175 - 25 + 1 = 151$ multiples of 4.

Step 4: Calculate the number of multiples of both 3 and 4 (i.e., multiples of 12). The smallest multiple of 12 is 108 (12×9) and the largest is 696 (12×58), resulting in $58 - 9 + 1 = 50$ multiples of 12.

Step 5: Use the principle of Inclusion-Exclusion to find the number of numbers that are multiples of 3 or 4. It is $200 + 151 - 50 = 301$.

Step 6: Subtract this from the total number of integers to find those that are neither multiples of 3 nor 4. The result is $601 - 301 = 300$.

COMMON TRAPS

- **Treating the interval endpoints as excluded.** For all integers from a to b inclusive, count $= b - a + 1$. *Avoid it:* Write the first and last allowed values explicitly: 100, 101, ..., 700; both endpoints are present.
- **Misreading 'neither ... nor' as excluding only common multiples.** Neither multiple of 3 nor multiple of 4 means not divisible by 3 AND not divisible by 4; complement is divisible by 3 OR divisible by 4. *Avoid it:* Translate the statement symbolically before calculating: required $=$ total $- |M_3 \cup M_4|$.

REVISE Interval notation and integer counting formulas. · Practice translating verbal set conditions into complements/unions/intersections.

34 Answer: B

Ref 10002

Medium ~3 min Functions

Fix the values of $f(1)$ and $f(3)$ first, then fill in the other three inputs.

Step 1. Pairs of distinct elements of B with sum 14 are $2 + 12$ and $4 + 10$. The pair $7 + 7$ is not allowed, since a one-one map cannot send 1 and 3 to the same value.

Step 2. Each pair can be assigned in 2 orders, so $(f(1), f(3))$ has 4 choices.

Step 3. Two values of B are now used, leaving 5. The inputs 7, 9 and 11 must get distinct values from these 5, in $5 \times 4 \times 3 = 60$ ways.

Step 4. Total $= 4 \times 60 = 240$.

Answer: (B) 240

COMMON TRAPS

- **Counting the valid sum pairs as unordered instead of ordered.** Since $f(1)$ and $f(3)$ are attached to labelled inputs 1 and 3, $(2, 12)$ and $(12, 2)$ are different functions. *Avoid it:* After listing each sum pair, explicitly write both assignments: $f(1) = 2, f(3) = 12$ and $f(1) = 12, f(3) = 2$.
- **Allowing the pair (7,7) despite injectivity.** One-one means distinct domain elements must have distinct images, so $f(1) = f(3) = 7$ is impossible because $1 \neq 3$. *Avoid it:* For every candidate pair, check both conditions: sum 14 and two distinct elements of B .

REVISE Injective maps from finite sets using the formula ${}_nP_r$ when assigning images to labelled domain elements. · Practice converting unordered value pairs into ordered function assignments.

35 Answer: 3

Ref 10211

Medium ~5 min Relations

QUICK APPROACH Count R row by row, then count the pairs in R whose reverses are not in R .

List R , then add the reverse of every pair whose reverse is missing.

Step 1. For each x , y must satisfy $y \geq \frac{4x}{5}$. $x = 1: y \geq 0.8, 5$ values. $x = 2: y \geq 1.6, 4$ values. $x = 3: y \geq 2.4, 3$ values. $x = 4: y \geq 3.2, 2$ values. $x = 5: y \geq 4, 2$ values. So $m = 16$.

Step 2. So R holds the 5 pairs (x, x) , all 10 pairs with $x < y$,

and $(5, 4)$.

Step 3. For symmetry, each pair (x, y) with $x < y$ needs (y, x) . Only $(5, 4)$ is already present, so $10 - 1 = 9$ pairs must be added. Thus $n = 9$.

Step 4. $m + n = 16 + 9 = 25$.

Answer: (3) 25

COMMON TRAPS

- **Missing the boundary pair.** $(5, 4)$ is in R because $20 \leq 20$, so its reverse $(4, 5)$ needs no addition. *Avoid it:* Check the equality cases when the inequality is not strict.
- **Adding pairs twice.** Each missing reverse is one new element, so exactly one pair is added for each pair of R without its reverse. *Avoid it:* List the missing reverses explicitly.

REVISE The symmetric closure of a relation.

36 Answer: 4

Ref 10268

Medium ~3 min Functions and their Properties

QUICK APPROACH Start by finding the prime factors of 2310 to determine the domain set A , then evaluate the function $f(x)$ for each element to find its range.

Factorize 2310 as $2 \times 3 \times 5 \times 7 \times 11$, so $A = \{2, 3, 5, 7, 11\}$ with $|A| = 5$.

Compute $f(x) = \lfloor \log_2(x^2 + \lfloor x^3/5 \rfloor) \rfloor$ for each:

- For $x = 2$: $\lfloor 8/5 \rfloor = 1, 4 + 1 = 5, \lfloor \log_2 5 \rfloor = 2$.
- For $x = 3$: $\lfloor 27/5 \rfloor = 5, 9 + 5 = 14, \lfloor \log_2 14 \rfloor = 3$.
- For $x = 5$: $\lfloor 125/5 \rfloor = 25, 25 + 25 = 50, \lfloor \log_2 50 \rfloor = 5$.
- For $x = 7$: $\lfloor 343/5 \rfloor = 68, 49 + 68 = 117, \lfloor \log_2 117 \rfloor = 6$.
- For $x = 11$: $\lfloor 1331/5 \rfloor = 266, 121 + 266 = 387, \lfloor \log_2 387 \rfloor = 8$.

Range of f is $\{2, 3, 5, 6, 8\}$ (5 distinct values). Option 2 (24) likely arose from mistakenly taking $|\text{range}| = 4$ and computing $4!$ or a partial permutation. Number of one-to-one functions from a 5-element set to a 5-element set is $5! = 120$.

Thus, the answer is 120 (option 4).

COMMON TRAPS

- **Dropping 11 while factorising 2310.** $2310 = 2 \times 3 \times 5 \times 7 \times 11$, so A has 5 elements. *Avoid it:* Divide repeatedly by small primes until the quotient is 1.
- **Assuming the range has as many elements as the domain.** Two primes could give the same value of f , so each value must be computed. *Avoid it:* Work out $f(x)$ for every prime and count the distinct values.

REVISE Counting one-one functions between finite sets.

37 Answer: 24

Ref 10278

Medium ~3 min Functions

List the points of A , read off B , then count the one-one maps.

Step 1. $2x + 3y = 23$ with $x, y \geq 1$ needs $23 - 3y$ to be even and positive, so y is odd and $y \leq 7$.

Step 2. $y = 1, 3, 5, 7$ give $x = 10, 7, 4, 1$. So $A = \{(10, 1), (7, 3), (4, 5), (1, 7)\}$ has 4 elements.

Step 3. B is the set of first coordinates, $\{1, 4, 7, 10\}$, which also has 4 elements.

Step 4. A one-one function from a 4-element set to a 4-element set is an arrangement of the 4 values, so there are $4! = 24$ of them.

Answer: 24

COMMON TRAPS

- **Counting B as set of pairs instead of first coordinates.** B contains only x -coordinates of ordered pairs in $A: \{1, 4, 7, 10\}$. *Avoid it:* Circle the variable before the colon: $B = \{x: \dots\}$; only x is collected.
- **Using total functions count 4^4 instead of one-one count $4!$.** One-one/injective requires distinct images for distinct domain elements. When $|A| = |B| = 4$, count is $4P_4 = 4!$. *Avoid it:* After computing sizes, ask whether repetition of images is allowed; for one-one, no.

REVISE Set-builder notation and projection of relations. Practice finite function counting: total, injective, surjective, bijective.

38 Answer: 25

Ref 10423

Medium ~3 min Relations

QUICK APPROACH Group the ordered pairs from A and from B by their sums, then match equal sums.

Count the ordered pairs (a_1, a_2) and (b_1, b_2) by their sums, and multiply for each common sum.

Step 1. Sums $a_1 + a_2$ with $a_1, a_2 \in A$ (ordered): 8 in 2 ways, 9 in 4 ways, 10 in 2 ways, 12 in 1 way, 13 in 2 ways, 14 in 1 way. The sums 4, 5 and 6 cannot match any sum from B .

Step 2. Sums $b_1 + b_2$ with $b_1, b_2 \in B$: 8 in 1 way, 9 in 2 ways, 10 in 3 ways, 11 in 2 ways, 12 in 3 ways, 13 in 2 ways, 14 in 2 ways, 16 in 1 way.

Step 3. For each common sum, multiply the counts: sum 8 gives $2 \times 1 = 2$, sum 9 gives $4 \times 2 = 8$, sum 10 gives $2 \times 3 = 6$, sum 12 gives $1 \times 3 = 3$, sum 13 gives $2 \times 2 = 4$, sum 14 gives $1 \times 2 = 2$.

Step 4. The number of elements in R is $2 + 8 + 6 + 3 + 4 + 2 = 25$.

Answer: 25

COMMON TRAPS

- **Counting unordered pairs.** (a_1, a_2) and (a_2, a_1) give different elements of R when $a_1 \neq a_2$, so a sum such as $2 + 3$ counts twice. *Avoid it:* Count ordered pairs for every sum.

REVISE Counting relations on Cartesian products.

39 Answer: 1

Ref 10427

Medium ~2 min Relations

QUICK APPROACH Start by checking the definitions of reflexive, symmetric, and transitive relations, then test these properties using the given divisibility condition.

To determine the nature of the relation R on $\mathbb{Z} \times \mathbb{Z}$ where $(a, b)R(c, d)$ iff $5 \mid (ad - bc)$, first verify reflexivity. For any (a, b) , $a \cdot b - b \cdot a = 0$, which is divisible by 5. Thus, R is reflexive.

For symmetry, suppose $(a, b)R(c, d)$, so $ad - bc = 5k$ for some integer k . Then for $(c, d)R(a, b)$, we have $cb - da = -(ad - bc) = -5k$, also divisible by 5. Hence, R is symmetric.

For transitivity, consider the counterexample $(1, 0), (0, 0), (0, 1)$: $(1, 0)R(0, 0)$ since $1 \cdot 0 - 0 \cdot 0 = 0$; $(0, 0)R(0, 1)$ since $0 \cdot 1 - 0 \cdot 0 = 0$; but $(1, 0)R(0, 1)$ fails as $1 \cdot 1 - 0 \cdot 0 = 1$, not divisible by 5. (Option 3 mistakenly assumed transitivity holds, overlooking cases involving $(0, 0)$ which relates to all pairs.)

Thus, R is reflexive and symmetric but not transitive, which is option [1].

40 Answer: 1

Ref 10448

Medium ~4 min Sets and their representation

The total number of subsets of a finite set with k elements is 2^k . Given that the total number of subsets of A is 56 more than that of B ,

$$2^m = 2^n + 56$$

or

$$2^m - 2^n = 56.$$

Assuming $m > n$, factor as

$$2^n(2^{m-n} - 1) = 56 = 2^3 \times 7.$$

Here, $2^{m-n} - 1$ must be an odd positive integer dividing 7, so possible values are 1 or 7.

If $2^{m-n} - 1 = 1$, then $2^{m-n} = 2$, so $m - n = 1$, and $2^n = 56$, which is not a power of 2.

If $2^{m-n} - 1 = 7$, then $2^{m-n} = 8$, so $m - n = 3$, and $2^n = 8$, hence $n = 3, m = 6$.

Thus, the point $P(m, n) = (6, 3)$.

The distance from $P(6, 3)$ to $Q(-2, -3)$ is

$$\sqrt{(6 - (-2))^2 + (3 - (-3))^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10.$$

Option 3 (4) encodes the "ignoring signs in Q" mistake (treating Q as $(2, 3)$, yielding horizontal distance 4). Therefore, the correct option is [1] 10.

COMMON TRAPS

- **Using m and n instead of 2^m and 2^n .** A set with m elements has 2^m subsets. *Avoid it:* Write $2^m - 2^n = 56$ and factor out 2^n .
- **Sign error in the distance formula.** The distance from (m, n) to $(-2, -3)$ uses $m + 2$ and $n + 3$. *Avoid it:* Subtract the coordinates of Q with their signs, then square.

REVISE Power sets and the distance formula.

41 Answer: 44

Ref 10468

Medium ~3 min Sets and their representation

QUICK APPROACH Use the concept of counting functions with constraints and start by determining how many subsets of A contain a specific element a .

For each input, count the subsets of A that can be its value.

Step 1. $f(a)$ must be a subset of A that contains a . Each of the other 6 elements may be in or out, so there are $2^6 = 64$ choices for $f(a)$.

Step 2. The 7 inputs are chosen independently, so the number of functions is $64^7 = (2^6)^7 = 2^{42}$.

Step 3. Writing $2^{42} = m^n$ with $m, n \in \mathbb{N}$, the least possible base is $m = 2$, which gives $n = 42$. Other forms such as 4^{21} or 64^7 have a larger base.

Step 4. $m + n = 2 + 42 = 44$.

Answer: 44

COMMON TRAPS

- **Ignoring the condition $a \in f(a)$.** Fixing a inside $f(a)$ leaves only the other 6 elements free, giving 2^6 choices. *Avoid it:* Count the free elements for each input before using the power rule.
- **Choosing a larger base than needed.** 2^{42} can also be written as 64^7 , but the least base is 2. *Avoid it:* Write the count as a power of a prime to get the least base.

REVISE Counting functions into a power set.

42 Answer: 2

Ref 7288

Medium ~3 min Sets and their representation

Count medals per man: each medal total counts a man once per event he won.

Step 1. Let a, b, c be the numbers of men with medals in exactly one, exactly two and all three events. Here $c = 5$.

Step 2. All 60 men won something, so $a + b + c = 60$.

Step 3. The total number of medals is $48 + 25 + 18 = 91$. A man in exactly one event gives 1 medal, in two events 2, in three events 3, so $a + 2b + 3c = 91$.

Step 4. Subtract the two equations: $b + 2c = 31$.

Step 5. With $c = 5, b = 31 - 10 = 21$.

Answer: (2) 21

43 Answer: 4

Ref 7326

Medium ~3 min Relations

Test symmetry and transitivity of each relation with a direct argument or a counterexample.

Step 1. T is symmetric: if $a^2 - b^2 = k \in \mathbb{Z}$, then $b^2 - a^2 = -k \in \mathbb{Z}$.

Step 2. T is also transitive: if $a^2 - b^2$ and $b^2 - c^2$ are integers, their sum $a^2 - c^2$ is an integer.

Step 3. S is not symmetric. Take $a = -1, b = 10$: $2 + \frac{a}{b} =$

$1.9 > 0$, so $(a, b) \in S$. But $2 + \frac{b}{a} = -8 < 0$, so $(b, a) \notin S$.

Step 4. Check the options. (1) fails because T is transitive. (2) fails because S is not symmetric. (3) fails because T is transitive. (4) is true.

Answer: (4) T is symmetric but S is not

44 Answer: 1

Ref 7335

Medium ~2 min Relations

Let the given expression be $A = q \vee ((\sim q) \wedge p)$. Its negation is $\sim A = \sim [q \vee ((\sim q) \wedge p)]$. By De Morgan's law, $\sim (X \vee Y) = \sim X \wedge \sim Y$, so $\sim A = \sim q \wedge \sim ((\sim q) \wedge p)$. Next, $\sim ((\sim q) \wedge p) = \sim (\sim q) \vee \sim p = q \vee (\sim p)$ (again by De Morgan). Thus, $\sim A = \sim q \wedge (q \vee \sim p)$. Distributing gives $(\sim q \wedge q) \vee (\sim q \wedge \sim p)$. Since $\sim q \wedge q \equiv F$, this simplifies to $\sim q \wedge \sim p$, or $(\sim p) \wedge (\sim q)$. Option [3] encodes the common error of stopping after partial De Morgan application without distributing and absorbing the contradiction. Hence, the negation is equivalent to option [1].

COMMON TRAPS

- **Stopping after De Morgan without simplifying.** $\sim q \wedge (q \vee \sim p)$ still contains $\sim q \wedge q$, which is false; the result is $\sim p \wedge \sim q$. *Avoid it:* Distribute and remove any contradiction before matching options.
- **Negating only one part.** The negation of $X \vee Y$ is $\sim X \wedge \sim Y$; both parts are negated. *Avoid it:* Check the answer on one row of the truth table.

REVISE De Morgan's laws and absorption.

45 Answer: 2

Ref 7370

Medium ~2 min Functions

QUICK APPROACH Check whether the given statement can ever be false, then look for an assignment that makes each option false.

Show that the given statement is a tautology, then find the option that is not one.

Step 1. $B \Rightarrow ((\sim A) \vee B)$ could fail only with B true and $(\sim A) \vee B$ false. But B true makes $(\sim A) \vee B$ true, so the statement is a tautology.

Step 2. Option (1): if B is true, $A \Rightarrow B$ is true, so $B \Rightarrow (A \Rightarrow B)$ is a tautology. Option (4) works the same way, since B true makes $(\sim A) \Rightarrow B$ true.

Step 3. Option (3): if A is true, $\sim A$ is false, so $(\sim A) \Rightarrow B$ is true. Hence $A \Rightarrow ((\sim A) \Rightarrow B)$ is a tautology.

Step 4. Option (2): take A true and B false. Then $A \Leftrightarrow B$ is false, so $A \Rightarrow (A \Leftrightarrow B)$ is false. It is not a tautology, so it is not equivalent to the given statement.

Answer: (2) $A \Rightarrow (A \Leftrightarrow B)$

COMMON TRAPS

- **Treating $\Leftrightarrow$ like $\Rightarrow$.** $A \Leftrightarrow B$ is false whenever A and B differ, so $A \Rightarrow (A \Leftrightarrow B)$ fails at A true and B false. *Avoid it:* Write the truth-table row of each connective separately.

REVISE Tautologies and logical equivalence of implications.

46 Answer: 2

Ref 7517

Medium ~4 min Composition of Functions

Write f as a formula in its own variable by expressing x in terms of $t = g(x)$.

Step 1. Let $t = \sqrt{x} + 1$. Then $\sqrt{x} = t - 1$ and $x = (t - 1)^2$.

Step 2. Substitute in $f(g(x)) = x + 3 - \sqrt{x}$: $f(t) = (t - 1)^2 + 3 - (t - 1)$.

Step 3. Expand: $f(t) = t^2 - 2t + 1 + 3 - t + 1 = t^2 - 3t + 5$.

Step 4. Put $t = 0$: $f(0) = 0 - 0 + 5 = 5$.

Step 5. Check at $x = 0$: $g(0) = 1$, and $f(1) = 1 - 3 + 5 = 3$, which equals $0 + 3 - 0$.

Answer: (2) 5

COMMON TRAPS

- **Treating the outer input of f as the same x in the composition.** Composition requires substituting the entire inner function into the outer function: $f(\sqrt{x} + 1) = x + 3 - \sqrt{x}$. *Avoid it:* Before evaluating $f(a)$, first solve $g(x) = a$, not $x = a$.
- **Using the step $g(0) = 1$ to answer $f(0)$.** Putting $x = 0$ in the composition gives $f(g(0)) = f(1)$, not $f(0)$. *Avoid it:* Write the actual argument of f after every composition substitution: $f(g(0)) = f(1)$.

REVISE For every composition problem, explicitly write $f(g(x))$ and label the argument of f . When using inverse substitution, first check whether the target value lies in the range of the inner function.

47 Answer: 6

Ref 7579

Medium ~6 min Relations

QUICK APPROACH Find the range of $2a + 3b$ first, then list only the values of $4c + 5d$ that fall in it.

List the values of $4c + 5d$ that $2a + 3b$ can reach, and count the matches.

Step 1. With $a, b \in \{1, 2, 3, 4\}$, $2a + 3b$ lies between 5 and 20. The values of $4c + 5d$ in that range are 9, 13, 14, 17, 18 and 19, from $(c, d) = (1, 1), (2, 1), (1, 2), (3, 1), (2, 2), (1, 3)$, one pair each.

Step 2. Solve $2a + 3b = k$ for each value. $k = 9$: $(3, 1)$. $k = 13$: $(2, 3)$. $k = 14$: $(4, 2)$ and $(1, 4)$. $k = 17$: $(4, 3)$. $k = 18$: $(3, 4)$. $k = 19$: none.

Step 3. Each match gives one element $((a, b), (c, d))$ of R , so the number of elements is $1 + 1 + 2 + 1 + 1 + 0 = 6$.

Answer: 6

COMMON TRAPS

- **Counting pairs instead of matches.** Each element of R is a pair of pairs, so a common value contributes (ways on the left) times (ways on the right). *Avoid it:* Tabulate both sides for each common value and multiply.

REVISE Counting solutions of linear equations in bounded integers.

48 Answer: 2039

Ref 7603

Hard ~4 min Relations, Functions

QUICK APPROACH Undo the inverse to find $f \circ g$, then compare it with $a(x^b + c) - 3$.

Invert the given inverse to get $f \circ g$, then match coefficients.

Step 1. If $y = \left(\frac{x-7}{2}\right)^{1/3}$, then $x = 2y^3 + 7$. So $(f \circ g)(x) = 2x^3 + 7$.

Step 2. Also $(f \circ g)(x) = a(x^b + c) - 3 = ax^b + ac - 3$. Matching gives $a = 2$, $b = 3$ and $ac - 3 = 7$, so $c = 5$.

Step 3. $ac = 10$, so $(f \circ g)(10) = 2 \times 1000 + 7 = 2007$.

Step 4. $f(b) = f(3) = 2 \times 3 - 3 = 3$, so $(g \circ f)(3) = g(3) = 3^3 + 5 = 32$.

Step 5. The sum is $2007 + 32 = 2039$.

Answer: 2039

COMMON TRAPS

- **Composing in the wrong order.** $(g \circ f)(b)$ means apply f first and then g , so it equals $g(f(b))$. *Avoid it:* Evaluate the inner function first in a composite.
- **Using the inverse as the function.** The given formula is $(f \circ g)^{-1}$, so it must be inverted before matching with $ax^b + ac - 3$. *Avoid it:* Solve $y = (f \circ g)^{-1}(x)$ for x first.

REVISE Composite functions and finding inverses.

49 Answer: A

Ref 7643

Medium ~5 min Sets and their representation

QUICK APPROACH List each event as ordered pairs (x, y) and count; for the last option use $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.

Count each event among the 36 outcomes (x, y) , with x on the 1st die and y on the 2nd.

Step 1. $A (x < y)$ has $\frac{36-6}{2} = 15$ outcomes. $B (x \text{ even}, y \text{ odd})$ has $3 \times 3 = 9$ outcomes, and $C (x \text{ odd}, y \text{ even})$ also has 9. So option (C) is false.

Step 2. $(2, 3)$ lies in both A and B , so A and B are not mutually exclusive. Option (B) is false.

Step 3. $B \cap C = \phi$, so $P(B \cap C) = 0$, while $P(B)P(C) = \frac{1}{4} \times \frac{1}{4} \neq 0$. So B and C are not independent, and option (D) is false.

Step 4. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C) = A \cap C$. An odd x less than an even y gives $(1, 2), (1, 4), (1, 6), (3, 4), (3, 6), (5, 6)$, which is 6 outcomes.

Answer: (A) the number of favourable cases of $(A \cup B) \cap C$ is 6

COMMON TRAPS

- **Confusing disjoint with independent.** Disjoint events with non-zero probabilities are never independent, because $P(B \cap C) = 0$ while $P(B)P(C) > 0$. *Avoid it:* Compare $P(B \cap C)$ with $P(B)P(C)$ as numbers.
- **Miscounting B.** An even first die and an odd second die allow 3 choices each, so B has 9 outcomes, not 6. *Avoid it:* Multiply the number of choices for each die.

REVISE Independent and mutually exclusive events.

50 Answer: 4

Ref 7653

Medium ~3 min Relations

Test each property with a specific pair of integers.

Step 1. Not reflexive: $\gcd(2, 2) = 2 \neq 1$, so $(2, 2) \notin R$.

Step 2. Not symmetric: $(2, 1)$ is in R , since $\gcd(2, 1) = 1$ and $2 \times 2 = 4 \neq 1$. But $(1, 2)$ is not, since $2 \times 1 = 2$ equals the second entry.

Step 3. Not transitive: $(2, 3)$ is in R ($\gcd = 1, 4 \neq 3$) and $(3, 4)$ is in R ($\gcd = 1, 6 \neq 4$). But $\gcd(2, 4) = 2$, so $(2, 4) \notin R$.

Step 4. So R is neither symmetric nor transitive. This rules out options (1), (2) and (3).

Answer: (4) neither symmetric nor transitive

51 Answer: 2

Ref 7671

Medium ~2 min Logical Connectives and Statements

QUICK APPROACH Rewrite $(p \vee q) \Rightarrow r$ as a conjunction of two implications, then compare it with each right side.

A biconditional is a tautology only if both sides agree in every row, so look for one row where they differ.

Step 1. $(p \vee q) \Rightarrow r$ is true exactly when both $p \Rightarrow r$ and $q \Rightarrow r$ are true.

Step 2. (S1): take $p = F, q = T, r = F$. Then $(p \vee q) \Rightarrow r$ is false, but $p \Rightarrow r$ is true. The two sides differ, so (S1) is not a tautology.

Step 3. (S2): take $p = T, q = F, r = F$. Then $(p \vee q) \Rightarrow r$ is false. Here $p \Rightarrow r$ is false and $q \Rightarrow r$ is true, so $(p \Rightarrow r) \vee (q \Rightarrow r)$ is true. The sides differ, so (S2) is not a tautology.

Step 4. Neither statement is a tautology.

Answer: (2) Neither (S1) nor (S2) is a tautology

COMMON TRAPS

- **Splitting with $\vee$ instead of $\wedge$.** $(p \vee q) \Rightarrow r$ needs both $p \Rightarrow r$ and $q \Rightarrow r$, so it equals their conjunction, not their disjunction. *Avoid it:* Test the split on the row $p = T, q = F, r = F$.

REVISE Standard equivalences of implications.

52 Answer: 4

Ref 7699

Medium ~4 min Greatest Integer Function

Let $n = \lfloor x \rfloor$, so $n \leq x < n + 1$ with $n \in \mathbb{Z}$. Rewrite the equation as

$$x^2 - 4x + n + 3 = nx \implies x^2 - (n + 4)x + (n + 3) = 0.$$

The discriminant is $(n+4)^2 - 4(n+3) = (n+2)^2$, so roots are

$$x = \frac{n+4 \pm (n+2)}{2},$$

giving $x = n+3$ or $x = 1$.

For $x = 1$, $[1] = 1$, so it holds only for $n = 1$. Substituting verifies: left side $1 - 4 + 1 + 3 = 1$, right side $1 \cdot 1 = 1$.

For $x = n+3$, it must lie in $[n, n+1)$, but $n+3 \geq n+3$ implies $n+3 < n+1$ or $3 < 1$, which is impossible.

Alternatively, factor: $x^2 - 4x + 3 + n(1-x) = (x-1)(x-3) - n(x-1) = (x-1)(x-3-n) = 0$. Thus $x = 1$ or $x - n = 3$. But $x - n = \{x\} \in [0, 1)$, so fractional part = 3 is impossible.

(Option 1 encodes the mistake of accepting both quadratic roots without interval checks.) Hence there is a unique solution $x = 1$ in $(-\infty, \infty)$, so option [4].

COMMON TRAPS

- **Cancelling the factor $(x-1)$ and losing a root.** With $[x] = n$ the equation becomes $(x-1)(x-3) = n(x-1)$, so $x = 1$ is a root in its own right. *Avoid it:* Factor instead of dividing, and keep each factor's root as a separate case.
- **Not checking the root against its interval.** The other root $x = n+3$ must satisfy $n \leq x < n+1$, which is impossible, so it gives no solution. *Avoid it:* After solving with $[x] = n$, always test $n \leq x < n+1$ for each candidate.

REVIEW Equations with the greatest integer function · Factorisation of quadratic equations

53 Answer: 19

Ref 7727

Medium ~4 min Relations

Every pair in R has $x > y$, so its reverse is never in R . Count the pairs in R .

Step 1. A has 5 even numbers $\{0, 4, 6, 8, 10\}$ and 3 odd numbers $\{3, 7, 9\}$.

Step 2. $x - y$ is a positive odd integer when x and y have opposite parity and $x > y$. Each of the $5 \times 3 = 15$ even-odd pairs gives exactly one such ordered pair.

Step 3. $x - y = 2$ gives $(6, 4)$, $(8, 6)$, $(10, 8)$ and $(9, 7)$: 4 more pairs. So R has 19 elements.

Step 4. For each $(x, y) \in R$, $y - x$ is negative, so $(y, x) \notin R$. A symmetric relation containing R must include all 19 reverse pairs.

Step 5. Adding exactly these 19 pairs makes the relation symmetric, so 19 is the minimum.

Answer: 19

54 Answer: 4

Ref 7812

Medium ~3 min Functions

QUICK APPROACH Complete the square in $x^2 - x + 1$ and note that both terms of f grow with it.

Let $g(x) = x^2 - x + 1$. Since g is always positive, $f = g + [g]$, and both parts are smallest where g is smallest.

Step 1. $g(x) = (x - \frac{1}{2})^2 + \frac{3}{4} \geq \frac{3}{4} > 0$, so $|g(x)| = g(x)$ and $f(x) = g(x) + [g(x)]$.

Step 2. On $[-1, 2]$, g takes its least value $\frac{3}{4}$ at $x = \frac{1}{2}$. There $[g] = 0$, so $f(\frac{1}{2}) = \frac{3}{4}$.

Step 3. At every other point $g(x) > \frac{3}{4}$ and $[g(x)] \geq 0$, so $f(x) > \frac{3}{4}$. For example, $f(-1) = 3 + 3 = 6$.

Step 4. The absolute minimum of f on $[-1, 2]$ is $\frac{3}{4}$.

Answer: (4) $\frac{3}{4}$

COMMON TRAPS

- **Rounding the minimum up.** At $g = \frac{3}{4}$ the greatest integer part is 0, not 1, so f equals $\frac{3}{4}$ there. *Avoid it:* Evaluate the greatest integer function carefully for values between 0 and 1.

REVIEW The greatest integer function and completing the square.

55 Answer: 1

Ref 7893

Medium ~3 min Relations

Test the three properties one by one, using counterexamples where they fail.

Step 1. Reflexive: $3a - 3a + \sqrt{7} = \sqrt{7}$ is irrational, so $(a, a) \in R$ for every a .

Step 2. Not symmetric: take $a = \frac{\sqrt{7}}{3}$, $b = 0$. Then $3a - 3b + \sqrt{7} = 2\sqrt{7}$ is irrational, but $3b - 3a + \sqrt{7} = 0$ is rational.

Step 3. Not transitive: take $a = \frac{\sqrt{7}}{3}$, $b = 1$, $c = \frac{2\sqrt{7}}{3}$. Then $2\sqrt{7} - 3$ and $3 - \sqrt{7}$ are irrational, so $(a, b), (b, c) \in R$, but $3a - 3c + \sqrt{7} = 0$, so $(a, c) \notin R$.

Answer: (1) Reflexive but neither symmetric nor transitive

COMMON TRAPS

- **Assuming the relation is symmetric.** Swapping a and b changes $3a - 3b$ to $3b - 3a$ but keeps $+\sqrt{7}$, so $a = \frac{\sqrt{7}}{3}$, $b = 0$ gives $2\sqrt{7}$ one way and 0 the other. *Avoid it:* Test symmetry with a pair where $3a - 3b$ is a multiple of $\sqrt{7}$.
- **Assuming the relation is transitive.** With $a = \frac{\sqrt{7}}{3}$, $b = 1$, $c = \frac{2\sqrt{7}}{3}$ both (a, b) and (b, c) give irrational values, yet $3a - 3c + \sqrt{7} = 0$ is rational. *Avoid it:* Look for a counterexample in which the irrational parts cancel only in the final step.

REVIEW Testing reflexive, symmetric and transitive properties with counterexamples.

56 Answer: 4

Ref 7933

Medium ~2 min Relations

The converse of $A \Rightarrow B$ is $B \Rightarrow A$, and any implication is equivalent to its contrapositive.

Step 1. Here $A = (\neg p) \wedge q$ and $B = r$, so the converse is $r \Rightarrow ((\neg p) \wedge q)$.

Step 2. This form is not among the options, so rewrite it by its

contrapositive: $r \Rightarrow A \equiv \neg A \Rightarrow \neg r$.

Step 3. By De Morgan's law, $\neg((\neg p) \wedge q) \equiv p \vee (\neg q)$.

Step 4. So the converse is equivalent to $(p \vee (\neg q)) \Rightarrow (\neg r)$.

Step 5. Option (3) has $\neg r$ in place of r , and options (1) and (2) change the connectives, so they are not equivalent to the converse.

Answer: (4) $(p \vee (\neg q)) \Rightarrow (\neg r)$

COMMON TRAPS

- **Confusing the converse with the contrapositive.** The converse of $A \Rightarrow B$ is $B \Rightarrow A$; its equivalent form is $\neg A \Rightarrow \neg B$. *Avoid it:* Name the converse, inverse and contrapositive before matching options.
- **Negating a conjunction incorrectly.** $\neg((\neg p) \wedge q) \equiv p \vee (\neg q)$ by De Morgan's law. *Avoid it:* Negate each part and change AND to OR.

REVISE Converse, inverse and contrapositive.

57 Answer: 3

Ref 7994

Medium ~3 min Sets and their representation

Count subsets of a set with $|A \times B|$ elements by their size.

Step 1. $|A \times B| = |A| \times |B| = 5 \times 2 = 10$.

Step 2. The number of subsets with k elements is $\binom{10}{k}$, so the count is $\binom{10}{3} + \binom{10}{4} + \binom{10}{5} + \binom{10}{6}$.

Step 3. These are 120, 210, 252 and 210.

Step 4. Their sum is $120 + 210 + 252 + 210 = 792$.

Step 5. Check: all subsets number $2^{10} = 1024$. Sizes 0 to 2 give $1 + 10 + 45 = 56$, and sizes 7 to 10 give $120 + 45 + 10 + 1 = 176$. Then $1024 - 56 - 176 = 792$.

Answer: (3) 792

58 Answer: 4

Ref 7998

Medium ~2 min Relations

Reduce the condition modulo 5 first; it then becomes equality of remainders.

Step 1. Since $3b \equiv -2b \pmod{5}$, we get $2a + 3b \equiv 2a - 2b \equiv 2(a - b) \pmod{5}$.

Step 2. As 2 and 5 are coprime, 5 divides $2(a - b)$ exactly when 5 divides $a - b$. So aRb means $a \equiv b \pmod{5}$.

Step 3. Reflexive: $a - a = 0$ is a multiple of 5.

Step 4. Symmetric: if 5 divides $a - b$, it divides $b - a$.

Step 5. Transitive: if 5 divides $a - b$ and $b - c$, it divides their sum $a - c$.

Step 6. So R is an equivalence relation.

Answer: (4) an equivalence relation

COMMON TRAPS

- **Assuming symmetry because $2a + 3b$ looks reversible.** Swapping gives $2b + 3a$, a different number; symmetry follows only after reducing to $a \equiv b \pmod{5}$. *Avoid it:* Reduce the condition modulo 5 before testing properties.
- **Adding the two conditions to prove transitivity.** $2a + 3b$ and $2b + 3c$ add to $2a + 5b + 3c$, which does not directly give $2a + 3c$. *Avoid it:* Use the simpler form $a \equiv b \pmod{5}$ instead.

REVISE Congruence modulo n as an equivalence relation.

59 Answer: 1

Ref 7999

Medium ~3 min Logic

Test each statement on the row where it could fail.

Step 1. For (S1), take p true and q false. Then $p \Rightarrow q$ is false and $(\neg p) \wedge q$ is false, so the disjunction is false. (S1) is not a tautology.

Step 2. For (S2), take p false and q true. Then $q \Rightarrow p$ is false, so the whole implication is true.

Step 3. A contradiction must be false in every row, so (S2) is not a contradiction.

Step 4. Both statements are false.

Answer: (1) Neither (S1) and (S2) is True

60 Answer: 3

Ref 8062

Medium ~4 min Relations

QUICK APPROACH Notice that $a_1 \leq b_2$ and $b_1 \leq a_2$ involve different coordinates, so count them separately.

The two conditions involve separate coordinates, so count each one and multiply.

Step 1. The condition $a_1 \leq b_2$ links $a_1 \in A$ with $b_2 \in B$. For $a_1 = 1, 3, 4, 6, 9$ the numbers of possible b_2 are 5, 4, 4, 2, 1, which gives 16 pairs.

Step 2. The condition $b_1 \leq a_2$ links $b_1 \in B$ with $a_2 \in A$. For $b_1 = 2, 4, 5, 8, 10$ the numbers of possible a_2 are 4, 3, 2, 1, 0, which gives 10 pairs.

Step 3. Each element of R is fixed by one pair (a_1, b_2) and one pair (b_1, a_2) , chosen independently. So the number of elements is $16 \times 10 = 160$.

Answer: (3) 160

COMMON TRAPS

- **Using a strict inequality.** The condition allows equality, so pairs such as $a_1 = b_2 = 4$ count. *Avoid it:* Read $\leq$ and $<$ carefully before counting.
- **Matching the wrong coordinates.** The first condition compares a_1 with b_2 , not with b_1 . *Avoid it:* Write which set each variable comes from before counting.

REVISE Counting elements of relations on Cartesian products.

61 Answer: A

Ref 8137

Medium ~2 min Logic and Statements

The statement "If I have fever, then I will take medicine and I will take rest" translates to logical form using the given

propositions. Since Q denotes "I will not take medicine," taking medicine is $\neg Q$. Thus, the statement is $P \rightarrow (\neg Q \wedge R)$.

An implication $A \rightarrow B$ is equivalent to $\neg A \vee B$, so $P \rightarrow (\neg Q \wedge R) \equiv \neg P \vee (\neg Q \wedge R)$.

Apply the distributive law: $X \vee (Y \wedge Z) \equiv (X \vee Y) \wedge (X \vee Z)$, with $X = \neg P$, $Y = \neg Q$, $Z = R$. This yields exactly $(\neg P \vee \neg Q) \wedge (\neg P \vee R)$, matching option [A].

Option [D], $(P \vee \neg Q) \wedge (P \vee R)$, simplifies to $P \vee (\neg Q \wedge R)$. This encodes the specific mistake of confusing the implication $A \rightarrow B$ with the disjunction $A \vee B$ (failing to negate the antecedent). Students may have been taught this error in translating conditional statements.

Verification by truth table or expansion confirms [A] matches in all cases, while [D] differs when P is false. Other options like [B] and [C] alter negations or connectives incorrectly. Thus, the equivalent form is option [A].

COMMON TRAPS

- **Forgetting that Q is already a negative statement.** Here Q means I will not take medicine, so I will take medicine is $\neg Q$. *Avoid it:* Translate each English clause into P , Q , R or its negation before writing any formula.
- **Writing an implication as P or the consequent.** $P \Rightarrow S$ is $\neg P \vee S$, not $P \vee S$. *Avoid it:* Always replace $P \Rightarrow S$ by $\neg P \vee S$ and then distribute over the AND.

REVISE Converting implications into disjunctions. · The distributive law for OR over AND.

62 Answer: 18

Ref 8145

Medium ~3 min Relations

QUICK APPROACH Substitute $t = a - b$ and solve $0 \leq 2t^2 + 3t \leq 4$ over the integers.

Put $t = a - b$ and find which integer values of t make $2t^2 + 3t$ lie in B .

Step 1. $2t^2 + 3t$ must be one of 0, 1, 2, 3, 4. Testing small integers: $t = 0$ gives 0 and $t = -2$ gives 2, while $t = -1, 1, 2, -3$ give $-1, 5, 14, 9$. Larger values of $|t|$ give larger results.

Step 2. So $t = 0$ or $t = -2$.

Step 3. $t = 0$ means $a = b$, which gives 10 pairs. $t = -2$ means $b = a + 2$ with $a, b \in A$, so $a = 1, 2, \dots, 8$, which gives 8 pairs.

Step 4. The number of elements in R is $10 + 8 = 18$.

Answer: 18

COMMON TRAPS

- **Missing negative differences.** $a - b$ can be negative, and $t = -2$ gives $2t^2 + 3t = 2$, which lies in B . *Avoid it:* Test negative values of t as well as positive ones.

REVISE Relations defined through the difference $a - b$.

63 Answer: 2

Ref 8151

Medium ~3 min Logic

Simplify the first implication, then test each choice of r in the second.

Step 1. $(p \wedge q) \Rightarrow (r \vee q)$ is always true: whenever $p \wedge q$ is true, q is true, so $r \vee q$ is true.

Step 2. So the whole statement is a tautology exactly when $(p \wedge r) \Rightarrow q$ is a tautology.

Step 3. $r = p$: this is $p \Rightarrow q$, false when p is true and q is false. Not a tautology.

Step 4. $r = q$: $(p \wedge q) \Rightarrow q$ is always true.

Step 5. $r = \neg p$: $p \wedge \neg p$ is always false, so the implication is always true.

Step 6. $r = \sim q$: $(p \wedge \sim q) \Rightarrow q$ is false when p is true and q is false. Not a tautology.

Step 7. The statement is a tautology for $r = q$ and $r = \neg p$, that is, for 2 values.

Answer: (2) 2

COMMON TRAPS

- **Assuming $r = p$ fails the second implication.** With $r = p$ the second part is $p \Rightarrow q$, which is false when p is true and q is false. *Avoid it:* Test each candidate for r on the row with p true and q false.
- **Checking only the first implication.** Both implications must hold in every row for the conjunction to be a tautology. *Avoid it:* Write a short truth table for each value of r and check both parts.

REVISE Building truth tables for compound statements.

64 Answer: 4

Ref 8169

Medium ~3 min Sets

The given statement is $\sim ((P \wedge Q) \vee (\sim P \wedge Q)) \Rightarrow ((\sim P) \wedge (\sim Q))$.

First simplify the subexpression inside the negation: $(P \wedge Q) \vee (\sim P \wedge Q)$. By the distributive law this factors as $(P \vee \sim P) \wedge Q \equiv T \wedge Q \equiv Q$. Thus its negation is $\sim Q$, and the original statement reduces to $\sim Q \Rightarrow (\sim P \wedge \sim Q)$.

Rewrite the implication: $\sim (\sim Q) \vee (\sim P \wedge \sim Q) \equiv Q \vee (\sim P \wedge \sim Q)$.

Distribute the disjunction: $Q \vee (\sim P \wedge \sim Q) \equiv (Q \vee \sim P) \wedge (Q \vee \sim Q) \equiv (\sim P \vee Q) \wedge T \equiv \sim P \vee Q$.

This matches option [4]. Option [2] $(\sim Q) \vee P$ encodes the error of mishandling distribution when converting the implication (or confusing it with the converse). Truth tables for the original and $\sim P \vee Q$ coincide in all cases, confirming equivalence.

COMMON TRAPS

- **Missing the simplification of the inner bracket.** $(P \wedge Q) \vee (\neg P \wedge Q)$ simplifies to Q , so the antecedent is $\neg Q$. *Avoid it:* Factor out the common Q before applying any negation.
- **Taking the contrapositive in the wrong direction.** $\neg Q \Rightarrow (\neg P \wedge \neg Q)$ is equivalent to $Q \vee \neg P$, not to $\neg Q \vee P$. *Avoid it:* Convert the implication to $\neg A \vee B$ and simplify step by step.

REVISE Absorption and distributive laws in logic.

65 Answer: 14

Ref 8180

Medium ~4 min Functions

QUICK APPROACH Call $g'(1)$ and $g''(2)$ constants, write g in terms of them, and use the definitions to set up equations.

Write f with unknown constants, build g from it, and match the definitions.

Step 1. Let $a = g'(1)$ and $b = g''(2)$. Then $f(x) = x^2 + ax + b$, $f'(x) = 2x + a$ and $f''(x) = 2$.

Step 2. So $g(x) = f(1)x^2 + x(2x + a) + 2 = (f(1) + 2)x^2 + ax + 2$. This gives $g'(x) = 2(f(1) + 2)x + a$ and $g''(x) = 2(f(1) + 2)$.

Step 3. $a = g'(1) = 2(f(1) + 2) + a$ gives $f(1) = -2$. Then $b = g''(2) = 0$.

Step 4. $f(1) = 1 + a + b = -2$ gives $a = -3$. So $f(x) = x^2 - 3x$ and $g(x) = -3x + 2$.

Step 5. $f(4) = 4$ and $g(4) = -10$, so $f(4) - g(4) = 14$.

Answer: 14

COMMON TRAPS

- **Treating $g'(1)$ as a function.** $g'(1)$ and $g''(2)$ are numbers, so f is an ordinary quadratic with constant coefficients. *Avoid it:* Rename such values as constants before differentiating.

REVISE Functions defined through their own derivative values.

66 Answer: 2

Ref 8382

Medium ~5 min Logical Reasoning

Step 1: Consider the initial logical expression $(p \wedge (\neg q)) \Rightarrow (p \Rightarrow (\neg q))$.

Step 2: The implication $(A \Rightarrow B)$ is logically equivalent to $\neg A \vee B$. Applying this, the expression becomes $\neg(p \wedge \neg q) \vee (\neg p \vee \neg q)$.

Step 3: Apply De Morgan's law: $\neg(p \wedge \neg q)$ is equivalent to $\neg p \vee q$.

Step 4: Thus, the expression becomes $(\neg p \vee q) \vee (\neg p \vee \neg q)$.

Step 5: Simplifying, we have $\neg p \vee q \vee \neg q$, which simplifies further to $\neg p \vee T$ (where T is a tautology), resulting in a tautology overall.

Short Explanation: The expression simplifies to $\neg p \vee T$, a tautology.

67 Answer: 4

Ref 8406

Medium ~2 min Logical Reasoning

Write each implication as a disjunction and factor out the common consequent Q .

Step 1. $P \Rightarrow Q \equiv \neg P \vee Q$ and $R \Rightarrow Q \equiv \neg R \vee Q$.

Step 2. So the statement is $(\neg P \vee Q) \wedge (\neg R \vee Q)$.

Step 3. By the distributive law, this equals $(\neg P \wedge \neg R) \vee Q$.

Step 4. By De Morgan's law, $\neg P \wedge \neg R \equiv \neg(P \vee R)$. So the statement is $\neg(P \vee R) \vee Q \equiv (P \vee R) \Rightarrow Q$.

Step 5. Check with P true, R false and Q false: the original statement and $(P \vee R) \Rightarrow Q$ are both false, while $(P \wedge R) \Rightarrow Q$ is true. So option 2 is not equivalent.

Answer: (4) $P \vee R \Rightarrow Q$

68 Answer: 3

Ref 8448

Medium ~4 min Functions

A tautology is true for all truth values of p and q . Recall $p \rightarrow q \equiv \neg p \vee q$. Test each pair (Δ, ∇) via truth table or simplification.

For $\Delta = \vee, \nabla = \vee$:

$$(p \rightarrow q) \vee (p \vee q) \equiv (\neg p \vee q) \vee (p \vee q) \equiv \neg p \vee p \vee q \equiv T.$$

This is always true.

Option 2 ($\Delta = \vee, \nabla = \wedge$) encodes the specific mistake of choosing conjunction for ∇ . Then

$$(p \rightarrow q) \vee (p \wedge q)$$

is false when $p = T, q = F$ (both disjuncts false), so not a tautology.

The other two cases fail similarly: $\Delta = \wedge$ yields false in the $p = T, q = F$ row. Only option 3 works.

Thus, $\Delta = \vee, \nabla = \vee$ (option 3).

69 Answer: 1

Ref 8480

Medium ~4 min Sets and their representation

Let $n(E) = 75$, $n(H) = 40$, and $n(E \cup H) = 100$ since each person speaks at least one language. Then $n(E \cap H) = 75 + 40 - 100 = 15$. Thus only English speakers $\alpha = n(E) - n(E \cap H) = 75 - 15 = 60$ and only Hindi speakers $\beta = n(H) - n(E \cap H) = 40 - 15 = 25$. (Note: equivalently, $\alpha = 100 - 40 = 60$, $\beta = 100 - 75 = 25$.)

The ellipse equation is $25(\beta^2 x^2 + \alpha^2 y^2) = \alpha^2 \beta^2$. Dividing both sides by $\alpha^2 \beta^2$ yields

$$\frac{25x^2}{\alpha^2} + \frac{25y^2}{\beta^2} = 1,$$

or equivalently

$$\frac{x^2}{(\alpha/5)^2} + \frac{y^2}{(\beta/5)^2} = 1.$$

Substituting values, $a = 60/5 = 12$ (semi-major axis) and $b = 25/5 = 5$ (semi-minor axis). The eccentricity is

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{25}{144}} = \sqrt{\frac{119}{144}} = \frac{\sqrt{119}}{12}.$$

Option 2 ($\sqrt{117}/12$) encodes the specific mistake of using

$144 - 27$ (instead of $144 - 25$) for the numerator under the square root, likely from an arithmetic error with $\beta^2/25$ or the intersection size. Thus the correct option is [1] $\sqrt{119}/12$.

70 Answer: 2

Ref 8494

Medium ~2 min Functions

Let $n = [x]$, an integer satisfying $n \leq x < n + 1$. For $f(x) = \frac{1}{\sqrt{n^2 - 3n - 10}}$ to be defined, the radicand must satisfy $n^2 - 3n - 10 > 0$ (strict inequality, as the denominator cannot be zero).

Factorizing gives $(n - 5)(n + 2) > 0$, so $n < -2$ or $n > 5$. Since $n \in \mathbb{Z}$, the possible values are $n \leq -3$ or $n \geq 6$.

For each $n \leq -3$, $x \in [n, n + 1)$; the union over these integers is $(-\infty, -2)$. For $n \geq 6$, the union is $[6, \infty)$.

Verification at boundaries: when $n = -2$ or $n = 5$, the radicand equals zero, so these are excluded (e.g., $x \in [-2, -1)$ or $[5, 6)$ are invalid). Option 1 errs by excluding the valid interval $[-3, -2)$ (where $n = -3$ works) and including $(5, 6)$ (where $n = 5$ makes the denominator zero).

Thus, the domain is $(-\infty, -2) \cup [6, \infty)$, option [2].

71 Answer: 4

Ref 8545

Medium ~3 min Logic and Reasoning

To verify (S_1) : simplify $(p \Rightarrow q) \wedge (p \wedge (\neg q))$. Note that $p \Rightarrow q \equiv \neg p \vee q$, so the expression is $(\neg p \vee q) \wedge p \wedge \neg q$. This equals $p \wedge \neg q \wedge (\neg p \vee q)$. Distributing yields $(p \wedge \neg q \wedge \neg p) \vee (p \wedge \neg q \wedge q)$. Both disjuncts contain contradictions: $p \wedge \neg p$ and $q \wedge \neg q$. Thus it is always false, a contradiction. Equivalently, $p \wedge \neg q \equiv \neg(p \Rightarrow q)$, so (S_1) is $A \wedge \neg A$ (where $A = p \Rightarrow q$), a contradiction. Hence (S_1) holds.

For (S_2) : the disjunction is $(p \wedge q) \vee (\neg p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$, the OR of all four minterms over p, q . Grouping gives $[p \wedge (q \vee \neg q)] \vee [\neg p \wedge (q \vee \neg q)] \equiv p \vee \neg p \equiv T$, a tautology.

Both statements are true. Option 1 (only (S_2) true) encoded the mistake of claiming $(p \Rightarrow q) \wedge (p \wedge \neg q)$ is not a contradiction, perhaps by overlooking that $p \wedge \neg q = \neg(p \Rightarrow q)$.

Correct option: [4]

72 Answer: 2

Ref 8550

Medium ~4 min Sets and their representation

QUICK APPROACH Find where $[x] - x$ is zero and which values it takes elsewhere.

Find the domain and the range from the size of $[x] - x$.

Step 1. For every real x , $0 \leq [x] - x < 1$, and it is 0 exactly when x is an integer. The square root in the denominator must be positive, so $A = \mathbb{R} - \mathbb{Z}$.

Step 2. For non-integer x , $[x] - x$ takes every value in $(0, 1)$. So $\sqrt{[x] - x} \in (0, 1)$ and $f(x) \in (1, \infty)$. Hence $B = (1, \infty)$.

Step 3. $A \cap B$ is $(1, \infty)$ with the integers removed, which is $(1, \infty) - \mathbb{N}$. So (S_1) is true.

Step 4. $A \cup B$ contains $-\frac{1}{2}$, which is not in $(1, \infty)$. So (S_2) is false.

Answer: (2) only (S_1) is true

COMMON TRAPS

- **Treating the ceiling as the floor.** $[x]$ is the smallest integer at least x , so $[x] - x$ lies in $[0, 1)$. *Avoid it:* Test a value such as $x = 2.3$ to fix the sign.
- **Forgetting negative numbers in the domain.** The domain contains every non-integer, negative ones included, so the union is larger than $(1, \infty)$. *Avoid it:* Look for an element outside the claimed set before accepting a union.

REVISE Domain and range with integer-part functions.

73 Answer: 43

Ref 8556

Medium ~4 min Sets and their representation

Sort the elements by their remainder on division by 3 and count by cases.

Step 1. Remainder 0: $\{3\}$. Remainder 1: $\{1, 7, 10\}$. Remainder 2: $\{2, 5, 11\}$.

Step 2. If a subset takes i elements of remainder 1 and j of remainder 2, its sum is a multiple of 3 exactly when $i + 2j$ is a multiple of 3, that is when $i \equiv j \pmod{3}$.

Step 3. With $0 \leq i, j \leq 3$, the allowed pairs are $(0, 0)$, $(1, 1)$, $(2, 2)$, $(3, 3)$, $(0, 3)$ and $(3, 0)$.

Step 4. Their counts are $1 + 3 \times 3 + 3 \times 3 + 1 + 1 + 1 = 22$.

Step 5. The element 3 can be added or left out freely, so there are $2 \times 22 = 44$ such subsets, including the empty one.

Step 6. Removing the empty subset leaves $44 - 1 = 43$.

Answer: 43

COMMON TRAPS

- **Treating total subsets as equally split among 3 remainders.** Subset counts are uniform only under specific symmetry conditions. Here residues are 0: one element, 1: three elements, 2: three elements, so the exact count must be computed by generating functions or residue DP. *Avoid it:* Classify elements by residues modulo 3: $\{3\}$, $\{1, 7, 10\}$, $\{2, 5, 11\}$; then count combinations with $i + 2j \equiv 0 \pmod{3}$.
- **Subtracting/handling the empty subset at the wrong stage.** The empty subset is included in most subset-counting methods and has sum 0, which is a multiple of 3. Subtract exactly 1 only after exact inclusion count is found. *Avoid it:* Write "include empty?" beside the counting method before starting.

REVISE Subset-sum modulo counting using residue distributions. Practice generating functions modulo $x^3 - 1$ or small dynamic programming tables.

74 Answer: 2

Ref 8561

Medium ~2 min Functions

QUICK APPROACH Use the distributive law to absorb the $(\sim p)$ term, then apply De Morgan's law.

Simplify the statement first, then negate it.

Step 1. By the distributive law, $(p \wedge (\sim q)) \vee (\sim p) = (p \vee (\sim p)) \wedge (\sim q \vee 1) = \sim q$.

$$p)) \wedge ((\sim q) \vee (\sim p)).$$

Step 2. $p \vee (\sim p)$ is always true, so the statement equals $(\sim q) \vee (\sim p)$, which is $\sim (p \wedge q)$ by De Morgan's law.

Step 3. Its negation is therefore $p \wedge q$.

Step 4. Check the other options. Option (1) contains $p \vee (\sim p)$, so it is always true. Option (3) is false at $p = q = T$, where $p \wedge q$ is true. Option (4) contains $p \wedge (\sim p)$, so it is always false.

Answer: (2) $p \wedge q$

COMMON TRAPS

- **Negating only one part.** The negation of a disjunction is the conjunction of the negations, so both parts must be negated. *Avoid it:* Apply De Morgan's law to the whole statement, not term by term.

REVISE De Morgan's laws and the distributive laws of logic.

75 Answer: 4

Ref 8573

Medium ~3 min Logic

QUICK APPROACH Start by using the equivalence $\neg(X \Rightarrow Y) \equiv X \wedge \neg Y$ to negate the main implication, then simplify the inner implication using logical equivalences for $X \Rightarrow Y$.

Let $P \equiv (A \wedge (B \vee C)) \Rightarrow (A \vee B)$. The given statement is $P \Rightarrow A$. Its negation is $P \wedge \neg A$ (by $\neg(X \Rightarrow Y) \equiv X \wedge \neg Y$).

Now simplify P . An implication $X \Rightarrow Y \equiv \neg X \vee Y$, so

$$P \equiv \neg(A \wedge (B \vee C)) \vee (A \vee B) \equiv (\neg A \vee (\neg B \wedge \neg C)) \vee (A \vee B).$$

This reduces to $\neg A \vee A \vee (\neg B \wedge \neg C) \vee B$. Since $\neg A \vee A \equiv T$ (tautology) and $T \vee$ anything is T , P is a tautology.

Thus the original statement $P \Rightarrow A \equiv T \Rightarrow A \equiv A$. Its negation is therefore $\neg A$.

Option 2 ($B \vee \neg C$) encodes the common mistake of partially distributing De Morgan's laws or stopping after writing $\neg(A \wedge (B \vee C))$ without verifying the full antecedent is always true.

Hence the negation is equivalent to $\neg A$, which is option [4].

COMMON TRAPS

- **Negating only the final consequent.** The negation of $S \Rightarrow A$ is $S \wedge \neg A$, not just $\neg A$ alone without checking S . *Avoid it:* Write the negation of $S \Rightarrow A$ as $S \wedge \neg A$, then simplify S .
- **Not seeing that the inner implication is always true.** $A \wedge (B \vee C) \Rightarrow A \vee B$ holds in every row, so S is a tautology and $S \wedge \neg A$ is $\neg A$. *Avoid it:* Check whether an inner implication is a tautology before expanding it.

REVISE Negation of an implication. · Identifying tautologies quickly.

76 Answer: 2

Ref 8602

Medium ~4 min Functions

Substitute values that cycle among themselves under $x \mapsto \frac{1}{1-x}$.

Step 1. $x = 2: \frac{1}{1-2} = -1$, so $f(2) + f(-1) = 3$.

Step 2. $x = -1: \frac{1}{1+1} = \frac{1}{2}$, so $f(-1) + f(\frac{1}{2}) = 0$.

Step 3. $x = \frac{1}{2}: \frac{1}{1-1/2} = 2$, so $f(\frac{1}{2}) + f(2) = \frac{3}{2}$.

Step 4. Add the three equations: $2[f(2) + f(-1) + f(\frac{1}{2})] = 3 + 0 + \frac{3}{2} = \frac{9}{2}$. So $f(2) + f(-1) + f(\frac{1}{2}) = \frac{9}{4}$.

Step 5. Subtract the equation of Step 2: $f(2) = \frac{9}{4} - 0 = \frac{9}{4}$.

Answer: (2) $\frac{9}{4}$

COMMON TRAPS

- **Stopping after two substitutions.** Two equations hold three unknowns, so a third substitution at $x = \frac{1}{2}$ is needed to close the cycle. *Avoid it:* Keep substituting until the arguments repeat, then solve the system.
- **Adding the right sides without halving.** Adding the three equations counts each unknown twice, so the sum must be divided by 2. *Avoid it:* After adding cyclic equations, count how many times each term appears.

REVISE Functional equations with cyclic substitutions.

77 Answer: B

Ref 8605

Medium ~2 min Functions

QUICK APPROACH Start by rewriting the implication using the equivalence $r \rightarrow s \equiv \neg r \vee s$, then apply De Morgan's laws to find the negation.

To find the negation of $(p \rightarrow q) \rightarrow (q \rightarrow p)$, let $A = p \rightarrow q$ and $B = q \rightarrow p$. The negation of $A \rightarrow B$ is $A \wedge \neg B$, so this is $(p \rightarrow q) \wedge \neg(q \rightarrow p)$.

Now, $\neg(q \rightarrow p) = q \wedge \neg p$ (negation of implication). Thus, the expression is $(p \rightarrow q) \wedge (q \wedge \neg p)$.

Rewrite $p \rightarrow q$ as $\neg p \vee q$, yielding $(\neg p \vee q) \wedge q \wedge \neg p$.

By associativity and absorption, $(\neg p \vee q) \wedge \neg p = \neg p$, so overall it simplifies to $q \wedge \neg p$.

This matches option [B]. Option [C] $(\neg q) \wedge p$ encodes the common mistake of negating the converse instead or swapping the order in the implication negation rule without simplifying correctly. The correct option is [B].

78 Answer: A

Ref 8608

Medium ~3 min Range of Functions

The function is defined for $-2 \leq x \leq 3$, as both radicands must be nonnegative. To derive the range of the continuous function $f(x) = \sqrt{(3-x)} + \sqrt{(2+x)}$ on this closed interval, evaluate at endpoints and critical points. At $x = -2$, $f(-2) = \sqrt{5}$. At $x = 3$, $f(3) = \sqrt{5}$. Differentiating gives $f'(x) = -1/(2\sqrt{(3-x)}) + 1/(2\sqrt{(2+x)})$. Setting $f'(x) = 0$ yields $\sqrt{(3-x)} = \sqrt{(2+x)}$, so $x = 1/2$. Then $f(1/2) = 2\sqrt{(5/2)} = \sqrt{(4 \cdot 5/2)} = \sqrt{10}$.

Alternatively, set $y = f(x) > 0$ and square: $y^2 = 5 + 2\sqrt{(6+x-x^2)}$. Here, $6+x-x^2 = 25/4 - (x-1/2)^2$ ranges over $[0, 25/4]$, so $\sqrt{(6+x-x^2)}$ ranges over $[0, 5/2]$. Thus y^2 ranges over $[5, 10]$, and y ranges over $[\sqrt{5}, \sqrt{10}]$. As f is continuous, it attains every intermediate value.

Option C encodes the specific mistake of an incorrect upper

bound $\sqrt{13}$ (perhaps from miscalculating the max of $6 + x - x^2$ as 4, leading to $y^2 = 13$). The correct option is [A] $[\sqrt{5}, \sqrt{10}]$.

COMMON TRAPS

- **Taking the maximum at an endpoint.** The sum of the two roots is largest where the two radicands are equal, at $x = \frac{1}{2}$, giving $\sqrt{10}$. *Avoid it:* Square $f(x)$ and find where the product term $\sqrt{(3-x)(2+x)}$ is largest.
- **Using the union of the radical conditions.** Both $3-x \geq 0$ and $2+x \geq 0$ must hold, so the domain is $[-2, 3]$. *Avoid it:* Intersect the conditions from every square root before finding the range.

REVISE Range of functions with square roots.

79 Answer: 4

Ref 8618

Medium ~3 min Functions

QUICK APPROACH Express $f(2)$, $f(3)$ and $f(4)$ in terms of $f(1)$, then use $|f(n)| \leq 8$.

Use the condition to write every value in terms of $a = f(1)$, then apply the bound $|f(n)| \leq 8$.

Step 1. $n = 1$: $f(1) + f(2) = 1$, so $f(2) = 1 - a$.

Step 2. $n = 2$: $f(2) + \frac{1}{2}f(3) = 1$, so $f(3) = 2(1 - f(2)) = 2a$.

Step 3. $n = 3$: $f(3) + \frac{1}{3}f(4) = 1$, so $f(4) = 3(1 - 2a)$.

Step 4. All values are integers with $|f(n)| \leq 8$. From $|2a| \leq 8$, $|a| \leq 4$. From $|3 - 6a| \leq 8$, $-\frac{5}{6} \leq a \leq \frac{11}{6}$, so $a = 0$ or $a = 1$. Both also satisfy $|1 - a| \leq 8$.

Step 5. $a = 0$ gives the values 0, 1, 0, 3 and $a = 1$ gives 1, 0, 2, -3. So there are 2 functions.

Answer: (4) 2

COMMON TRAPS

- **Choosing every value freely.** Once $f(1)$ is chosen, the condition fixes all other values, so only the bounds limit the count. *Avoid it:* Write every value in terms of $f(1)$ before counting.

REVISE Functions defined by a recurrence on a finite set.

80 Answer: 1

Ref 8848

Medium ~3 min Relations

For R_1 , the condition AR_1B is $(A \cap B^c) \cup (B \cap A^c) = \phi$, i.e., the symmetric difference $A \Delta B = \phi$. This holds precisely when no element lies in exactly one of A or B , so $A = B$. Thus R_1 is the equality relation on $P(S)$. Equality is reflexive ($A = A$), symmetric (if $A = B$ then $B = A$), and transitive (if $A = B$ and $B = C$ then $A = C$), hence an equivalence relation.

For R_2 , the condition is $A \cup B^c = B \cup A^c$. First, it is reflexive: $A \cup A^c = A \cup A^c$ holds. It is symmetric because swapping A and B yields the same equality. To show it forces $A = B$, suppose there is $x \in A \setminus B$. Then $x \in A$ and $x \in B^c$, so $x \in A \cup B^c$. But $A \cup B^c = B \cup A^c$, hence $x \in B \cup A^c$. Since $x \notin B$, necessarily $x \in A^c$, contradicting $x \in A$. Similarly, if $y \in B \setminus A$ then $y \in B \cup A^c = A \cup B^c$ forces $y \in B^c$, again a contradiction. Thus $A = B$ (converse is immediate). So R_2 is also equality, an equivalence relation.

Option 2 encoded the specific mistake of assuming R_2 fails transitivity (or confuses the condition with inclusion). Both R_1 and R_2 are therefore equivalence relations. The correct option is [1].

81 Answer: 4

Ref 8862

Medium ~2 min Functions

Start with the given expression: $\neg(p \wedge q) \vee q$. Apply De Morgan's law to obtain $(\neg p \vee \neg q) \vee q$. By associativity, this is $\neg p \vee (\neg q \vee q)$. Since $\neg q \vee q \equiv T$ (tautology), the expression simplifies to $\neg p \vee T \equiv T$.

Now consider option [4]: $p \rightarrow (p \vee q) \equiv \neg p \vee (p \vee q)$. This rearranges to $(\neg p \vee p) \vee q \equiv T \vee q \equiv T$. Thus both are tautologies and equivalent.

Option [2] is $p \rightarrow q \equiv \neg p \vee q$, which is not a tautology (false when $p = T, q = F$). The error was likely stopping after partial simplification or confusing with $\neg(p \wedge q) \equiv \neg p \vee \neg q$ without absorbing the disjunct q . Options [1] and [3] also reduce to $\neg q \vee (p \wedge q)$ and $\neg p \vee q$, respectively, neither equivalent to T .

The expression is equivalent to option [4].

82 Answer: B

Ref 8439

Medium ~3 min Union, intersection and complement

Use inclusion-exclusion on the 3-digit numbers, then remove the multiples of 48.

Step 1. Multiples of 3 from 100 to 999: $333 - 33 = 300$.

Step 2. Multiples of 4: $249 - 24 = 225$.

Step 3. Multiples of both, that is of 12: $83 - 8 = 75$.

Step 4. Divisible by 3 or 4: $300 + 225 - 75 = 450$.

Step 5. Every multiple of 48 is a multiple of 3 and of 4, so all of them lie inside these 450. The 3-digit multiples of 48 run from $48 \times 3 = 144$ to $48 \times 20 = 960$, which is 18 numbers.

Step 6. Required count = $450 - 18 = 432$.

Answer: (B) 432

83 Answer: 4

Ref 7825

Medium ~3 min Relations

Simplify each compound statement with the rule $p \Rightarrow q \equiv \sim p \vee q$.

Step 1. For S_1 , $p \Rightarrow q \equiv \sim p \vee q$, and $p \wedge \sim q$ is its negation, since $\sim(\sim p \vee q) \equiv p \wedge \sim q$.

Step 2. So S_1 reads $X \vee \sim X$ with $X = (p \Rightarrow q)$. A statement or its negation is always true, so S_1 is a tautology.

Step 3. For S_2 , $\sim p \Rightarrow \sim q \equiv p \vee \sim q$ and $\sim p \vee q \equiv p \Rightarrow q$. The conjunction $(p \vee \sim q) \wedge (\sim p \vee q)$ equals $(q \Rightarrow p) \wedge (p \Rightarrow q) \equiv p \Leftrightarrow q$.

Step 4. $p \Leftrightarrow q$ is true when p and q are both true, so S_2 is not a contradiction. S_2 is wrong.

Answer: (4) only S1 is correct

COMMON TRAPS

- **Calling a biconditional a contradiction.** $(p \vee \sim q) \wedge (\sim p \vee q) \equiv p \Leftrightarrow q$, which is true when $p = q = T$. *Avoid it:* Find one true row before calling a statement a contradiction.
- **Missing that S1 is $X \vee \sim X$.** $p \wedge \sim q$ is the negation of $p \Rightarrow q$, so S1 is always true. *Avoid it:* Check whether one part is the negation of the other.

REVISE Tautologies, contradictions and the biconditional.

84 Answer: 2

Ref 9018

Medium ~2 min Relations

QUICK APPROACH Translate the sentence into $(A \wedge C) \rightarrow B$ and recall when an implication is false.

Write the sentence in symbols, then negate the implication.

Step 1. 'Rishi is a judge' is A and 'he is not arrogant' is C , so the statement is $(A \wedge C) \rightarrow B$.

Step 2. The negation of $P \rightarrow Q$ is $P \wedge (\sim Q)$, because an implication is false only when P is true and Q is false.

Step 3. So the negation is $(A \wedge C) \wedge (\sim B)$, which is $(\sim B) \wedge (A \wedge C)$.

Step 4. Options (1), (3) and (4) are implications with premise B , so each is true whenever B is false, even with A false. The negation is false in that case, so these options differ from it.

Answer: (2) $(\sim B) \wedge (A \wedge C)$

COMMON TRAPS

- **Writing $\sim C$ for 'not arrogant'.** Here C already means 'Rishi is not arrogant', so the phrase 'he is not arrogant' is C itself. *Avoid it:* Match each phrase to the letters exactly as defined.
- **Negating into another implication.** The negation of $P \rightarrow Q$ is $P \wedge (\sim Q)$, not $Q \rightarrow P$ or $(\sim P) \rightarrow (\sim Q)$. *Avoid it:* Remember that an implication fails only when P is true and Q is false.

REVISE Negation of conditional statements.

85 Answer: B

Ref 6121

Medium ~4 min Relations

For each allowed b , the first entry a can be any of the 60 numbers, so count the possible values of b .

Step 1. $b = pq$ with p, q odd primes (at least 3) and $b \leq 60$.

Step 2. With $p = 3$: $q = 3, 5, 7, 11, 13, 17, 19$ give $b = 9, 15, 21, 33, 39, 51, 57$. Next, $3 \times 23 = 69$ is too big.

Step 3. With $p = 5$ and $q \geq 5$: $b = 25, 35, 55$. With $p = 7$ and $q \geq 7$: $b = 49$. Larger p gives $p^2 > 60$.

Step 4. So there are $7 + 3 + 1 = 11$ values of b , all different.

Step 5. Each pairs with 60 choices of a , so $n(R) = 60 \times 11 = 660$.

Answer: (B) 660

86 Answer: C

Ref 6126

Medium ~2 min Logical statements and truth tables

Step 1: Understand each option and determine if it is a tautology.

Option A: $((\sim q) \wedge p) \wedge q$ - This is not a tautology because there are possible truth value assignments (e.g., $q = \text{false}$) that make the expression false.

Option B: $((\sim q) \wedge p) \wedge (p \wedge (\sim p))$ - This expression will always be false because $(p \wedge (\sim p))$ is always false (contradiction).

Option C: $((\sim q) \wedge p) \vee (p \vee (\sim p))$ - This is a tautology. The second part $(p \vee (\sim p))$ is always true regardless of p 's value. Hence, the entire expression is always true.

Option D: $(p \wedge q) \wedge (\sim (p \wedge q))$ - This expression is never true, as $(p \wedge q)$ and its negation cannot both be true simultaneously.

Step 2: Identify the tautology, which is option C.

COMMON TRAPS

- **Missing a contradiction hidden by associativity.** $((\sim q) \wedge p) \wedge q$ contains both q and $\sim q$ joined by AND, so it is always false. *Avoid it:* Flatten repeated ANDs and look for a letter with its negation.
- **Thinking a tautology must have no false part.** $X \vee (p \vee \sim p)$ is always true whatever X is, because one part is always true. *Avoid it:* Look for a part like $p \vee \sim p$ joined by OR.

REVISE Tautologies and contradictions.

87 Answer: 4

Ref 6128

Medium ~3 min Relations

" p only if X " means p cannot happen without X , which is $p \Rightarrow X$.

Step 1. "He is in his village" is the negation of q , so it is $\sim q$.

Step 2. "It is Sunday or Saturday" is $r \vee s$.

Step 3. The condition is "in his village and (Sunday or Saturday)", which is $(\sim q) \wedge (r \vee s)$.

Step 4. Ramesh listens to music only if this condition holds, so the statement is $p \Rightarrow ((\sim q) \wedge (r \vee s))$.

Step 5. Options (1) and (2) reverse the direction of the implication, and option (3) uses q (out of the village) instead of $\sim q$.

Answer: (4) $p \Rightarrow ((\sim q) \wedge (r \vee s))$

COMMON TRAPS

- **Reversing the direction of “only if”.** “P only if C” means P cannot happen unless C is true, so the correct implication is $P \Rightarrow C$, not $C \Rightarrow P$. *Avoid it:* Rewrite “P only if C” as “If P, then C” before looking at options.
- **Using q for “in his village” instead of negating it.** Since q means “out of his village”, “in his village” is the negation of q, written $\neg q$ or $\neg q$. *Avoid it:* Underline direction words like “out of”, “not”, “inside”, “outside” in the proposition definitions.

REVISE Memorize the translation pair: “P if C” = $C \Rightarrow P$, while “P only if C” = $P \Rightarrow C$. · For every proposition definition, write its negation in words before solving.

88 Answer: 4

Ref 6189

Medium ~2 min Relations

The set A is partitioned into disjoint A_i whose union is A . The relation is defined by $R = \{(x, y) : y \in A_i \text{ iff } x \in A_i, 1 \leq i \leq k\}$. This means $(x, y) \in R$ precisely when x and y lie in exactly the same block A_m of the partition.

Reflexivity: Let $x \in A_m$. Setting $y = x$ satisfies $y \in A_m$ iff $x \in A_m$, so $(x, x) \in R$ for every x .

Symmetry: Suppose $(x, y) \in R$. Then $x, y \in A_m$ for the same m , which is identical to the condition for (y, x) , so $(y, x) \in R$.

Transitivity: Let $(x, y) \in R$ and $(y, z) \in R$. Then $x, y \in A_m$ and $y, z \in A_n$. The disjointness $A_m \cap A_n = \emptyset$ for $m \neq n$ forces $m = n$, hence $x, z \in A_m$ and $(x, z) \in R$.

Thus R is reflexive, symmetric and transitive. Option 3 encodes the specific mistake of denying symmetry and transitivity, perhaps from misreading the biconditional as an exclusive-or or from confusing R with the equality relation on A . The correct choice is therefore option 4: an equivalence relation.

89 Answer: B

Ref 6409

Medium ~3 min Logic and Propositions

To determine for which $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ the formula $(p \Delta q) \Rightarrow [(p \Delta \neg q) \vee (\neg p \Delta q)]$ is a tautology, evaluate when the implication can fail: only if the antecedent is true while the consequent is false. Use truth table checks or algebraic simplification for each operator.

Case $\Delta = \wedge$: Antecedent is $p \wedge q$. Consequent simplifies to $(p \wedge \neg q) \vee (\neg p \wedge q)$, i.e., $p \oplus q$. At $p = T, q = T$, antecedent = T but consequent = F, so $T \Rightarrow F$ evaluates to F. Not a tautology. (This is the decisive counterexample.)

Case $\Delta = \vee$: Antecedent is $p \vee q$. Consequent is $(p \vee \neg q) \vee (\neg p \vee q) = p \vee \neg p \vee q \vee \neg q \equiv T$. Since consequent is always true, the implication is a tautology.

Case $\Delta = \Rightarrow$: Antecedent is $\neg p \vee q$. Consequent is $(\neg p \vee \neg q) \vee (p \vee q) = p \vee \neg p \vee q \vee \neg q \equiv T$. Again, implication is a tautology.

Case $\Delta = \Leftrightarrow$: Antecedent is $p \leftrightarrow q$. Both $(p \leftrightarrow \neg q)$ and $(\neg p \leftrightarrow q)$ equal $p \oplus q$, so consequent = $p \oplus q$. At $p = q$ (both T or both F), antecedent = T but consequent = F, so implication = F. Not a tautology.

Only $\vee$ and $\Rightarrow$ work, hence two choices. Option D (4) encodes the error of assuming the formula holds for all four operators without checking cases where antecedent is true. The correct option is [B].

COMMON TRAPS

- **Counting $\wedge$ as valid despite the TT counterexample.**

For $\Delta = \wedge$, at $(p, q) = (T, T)$, $(p \wedge q) = T$ but $(p \wedge \neg q) \vee (\neg p \wedge q) = F \vee F = F$, so the whole implication is false. *Avoid it:* For each Δ , first test rows where $p \Delta q$ is true; those are the only rows that can break $A \Rightarrow B$.

- **Counting $\Leftrightarrow$ as valid by treating negation as preserving equivalence.**

For $\Delta = \Leftrightarrow$, $\text{RHS} = (p \Rightarrow \neg q) \vee (\neg p \Rightarrow q) = \neg(p \Rightarrow q)$, hence the formula is $A \Rightarrow \neg A$, not tautological. *Avoid it:* Check the same-value rows (T, T) and (F, F), where $p \Leftrightarrow q$ is true.

REVISE Use a full 4-row truth table whenever the answer choices count tautological connectives. · For implication $A \Rightarrow B$, search only for rows with $A = T$ and $B = F$.

90 Answer: B

Ref 6418

Medium ~3 min Functions

QUICK APPROACH Start by applying the concept of counting functions and use the constraint $f(1) + f(2) = f(3)$ to determine valid triples $(f(1), f(2), f(3))$ within the codomain.

Choose $f(1)$ and $f(2)$ so that their sum is a value in the codomain; then $f(3)$ is fixed and $f(4)$ is free.

Step 1. Let $a = f(1)$ and $b = f(2)$. Then $f(3) = a + b$ must lie in $\{1, \dots, 6\}$, so $2 \leq a + b \leq 6$.

Step 2. Count pairs by a : $a = 1$ gives $b = 1$ to 5 (5 pairs), $a = 2$ gives 4, $a = 3$ gives 3, $a = 4$ gives 2, $a = 5$ gives 1, and $a = 6$ gives none.

Step 3. That is $5 + 4 + 3 + 2 + 1 = 15$ pairs, and each fixes $f(3)$.

Step 4. $f(4)$ can be any of the 6 values.

Step 5. Total number of functions = $15 \times 6 = 90$.

Answer: (B) 90

91 Answer: D

Ref 6504

Medium ~3 min Relations

An implication is false only when its left side is true and its right side is false. Use this to fix P, Q, R , then test each option.

Step 1. $P \wedge \neg R = T$ gives $P = T$ and $R = F$.

Step 2. $\neg R \wedge Q = F$ with $\neg R = T$ gives $Q = F$.

Step 3. Option (A): $P \vee Q = T$ and $\neg R = T$, so $T \rightarrow T = T$.

Step 4. Option (B): $R \vee Q = F$, so $F \rightarrow \neg P$ is T.

Step 5. Option (C): $\neg(P \vee Q) = F$, so the implication is T.

Step 6. Option (D): $\neg(R \vee Q) = \neg F = T$ and $\neg P = F$, so $T \rightarrow F = F$.

Answer: (D) $\neg(R \vee Q) \rightarrow \neg P$

92 Answer: Marks to all

Ref 6514

Medium ~3 min Sets and their representation

QUICK APPROACH Treat S_n and T_n as circles and compare the distance between their centres with the sum and the difference of the radii.

S_n and T_n are circles, and two circles do not meet when the centre distance is more than the sum of the radii or less than their difference.

Step 1. S_n has centre $(3, -2)$ and radius $\frac{n}{4}$. T_n has centre $(2, -3)$ and radius $\frac{1}{n}$. The centre distance is $d = \sqrt{2} \approx 1.414$.

Step 2. The circles lie outside each other when $\frac{n}{4} + \frac{1}{n} < \sqrt{2}$. The sums for $n = 1, 2, 3, 4$ are 1.25, 1, 1.08, 1.25, so these four work. For $n \geq 5$ the sum is more than $\sqrt{2}$.

Step 3. One circle lies inside the other when $\frac{n}{4} - \frac{1}{n} > \sqrt{2}$. For $n = 5$ and 6 the differences are 1.05 and 1.33, so those circles meet. For every $n \geq 7$ the difference is at least 1.61, so the circles do not meet.

Step 4. The set is $\{1, 2, 3, 4\} \cup \{7, 8, 9, \dots\}$, which is infinite, so none of the four options is correct.

NTA dropped this question and gave marks to all candidates.

Answer: no option fits; marks to all candidates

COMMON TRAPS

- **Checking only outside circles.** Two circles also fail to meet when one lies wholly inside the other, which happens when the centre distance is less than the difference of the radii. *Avoid it:* Test both $d > r_1 + r_2$ and $d < |r_1 - r_2|$ before counting.

REVISE The relative positions of two circles.

93 Answer: 4

Ref 6516

Medium ~3 min Logic and Propositions

Simplify the statement, then simplify the options and compare.

Step 1. $p \Leftrightarrow \sim q$ is true exactly when p and q differ, so it is the negation of $p \Leftrightarrow q$. Hence $\sim(p \Leftrightarrow \sim q) \equiv p \Leftrightarrow q$.

Step 2. The statement is $(p \Leftrightarrow q) \wedge q$. When $q = T$, $p \Leftrightarrow q$ is just p ; when $q = F$ the whole is false. So it equals $p \wedge q$.

Step 3. $p \wedge q$ is false for $p = F$, $q = T$ and true for $p = q = T$, so it is neither a tautology nor a contradiction.

Step 4. Option (3): $(p \Rightarrow q) \wedge q \equiv q$, which differs from $p \wedge q$ when $p = F$, $q = T$.

Step 5. Option (4): $(p \Rightarrow q) \wedge p \equiv (\sim p \vee q) \wedge p \equiv p \wedge q$. This matches.

Answer: (4) equivalent to $(p \Rightarrow q) \wedge p$

COMMON TRAPS

- **Calling the statement a contradiction.** It equals $p \wedge q$, which is true when $p = q = T$, so it is not a contradiction. *Avoid it:* Find one true row before calling a statement a contradiction.
- **Mishandling negation of a biconditional.** $\sim(p \Leftrightarrow \sim q) \equiv p \Leftrightarrow q$, since $p \Leftrightarrow \sim q$ is true exactly when p and q differ. *Avoid it:* Check a biconditional rule on all four rows.

REVISE The biconditional and its negation.

94 Answer: 107

Ref 6520

Medium ~4 min Sets and their representation

QUICK APPROACH Count B through its complement, then add only the prime-sum subsets that are not already in B .

Split $B \cup C$ into B and the part of C that lies outside B .

Step 1. $T \notin B$ exactly when $1 \in T$ and $2 \notin T$. There are $2^5 = 32$ such subsets, so $|B| = 128 - 32 = 96$.

Step 2. A subset outside B is $\{1\} \cup U$ with $U \subseteq \{3, 4, 5, 6, 7\}$, and its sum is $1 + s$, where s is the sum of U . Since $s \leq 25$, a prime $1 + s$ needs $s \in \{1, 2, 4, 6, 10, 12, 16, 18, 22\}$.

Step 3. Count the sets U . $s = 4$: $\{4\}$. $s = 6$: $\{6\}$. $s = 10$: $\{3, 7\}, \{4, 6\}$. $s = 12$: $\{5, 7\}, \{3, 4, 5\}$. $s = 16$: $\{3, 6, 7\}, \{4, 5, 7\}$. $s = 18$: $\{5, 6, 7\}, \{3, 4, 5, 6\}$. $s = 22$: $\{4, 5, 6, 7\}$. The sums 1 and 2 cannot occur. That gives 11 subsets.

Step 4. So $|B \cup C| = 96 + 11 = 107$.

Answer: 107

COMMON TRAPS

- **Adding all of C .** Subsets in both B and C would be counted twice, so only the prime-sum subsets outside B are added. *Avoid it:* Use $|B \cup C| = |B| + |B' \cap C|$.
- **Misreading the condition for B .** The condition either $1 \notin T$ or $2 \in T$ fails only when $1 \in T$ and $2 \notin T$. *Avoid it:* Negate the condition first and count the subsets that fail it.

REVISE Counting subsets with complements and inclusion-exclusion.

95 Answer: 3

Ref 6555

Medium ~3 min Relations

Simplify both statements first; if they are equivalent, they always have the same truth value.

Step 1. $S_1 = (\neg p \vee q) \vee (\neg p \vee r) \equiv \neg p \vee q \vee r$, by associativity and $\neg p \vee \neg p \equiv \neg p$.

Step 2. $S_2 = p \rightarrow (q \vee r) \equiv \neg p \vee q \vee r$.

Step 3. So $S_1 \equiv S_2$: in every row they are both true or both false.

Step 4. Statements (1), (2) and (4) each say that one has the same truth value as the other, so they are true.

Step 5. Statement (3) says that when S_2 is false, S_1 is true. This never happens, because S_1 is then false too.

Answer: (3) If S_2 is False, then S_1 is True

COMMON TRAPS

- **Not simplifying before testing the statements.** Both S_1 and S_2 reduce to $\neg p \vee q \vee r$, so they always agree. *Avoid it:* Simplify each statement fully before reading the options.
- **Reading a NOT-true question as asking for a true statement.** The question asks for the statement that fails, which is the one claiming different truth values. *Avoid it:* Underline NOT in the question before choosing.

REVIEW Logical equivalence of compound statements.

96 Answer: 4

Ref 6564

Medium ~3 min Logic and Reasoning

Rewrite the implication as a disjunction and simplify.

Step 1. $(p \wedge q) \Rightarrow (p \wedge r) \equiv \neg(p \wedge q) \vee (p \wedge r) \equiv \neg p \vee \neg q \vee (p \wedge r)$.

Step 2. By the distributive law, $\neg p \vee (p \wedge r) \equiv (\neg p \vee p) \wedge (\neg p \vee r) \equiv \neg p \vee r$.

Step 3. So the statement is $\neg p \vee \neg q \vee r$.

Step 4. Option (4): $(p \wedge q) \Rightarrow r \equiv \neg p \vee \neg q \vee r$. This matches.

Step 5. The others differ. Option (1) is $\neg q \vee (p \wedge r)$, false for $p = F, q = T$. Option (2) is $\neg p \vee r$, false for $p = T, r = F$. Option (3) is false for $p = T, q = F, r = T$.

Answer: (4) $(p \wedge q) \Rightarrow r$

97 Answer: 2

Ref 6566

Medium ~3 min Relations

Check reflexive, symmetric and transitive for each relation; one counterexample is enough to fail.

Step 1. $R_1: |a - a| = 0 \leq 13$, so it is reflexive, and $|a - b| = |b - a|$ makes it symmetric.

Step 2. R_1 is not transitive: (1, 10) and (10, 20) are in R_1 since the differences are 9 and 10, but $|1 - 20| = 19 > 13$.

Step 3. $R_2: |a - a| = 0 \neq 13$, so it is reflexive, and it is symmetric for the same reason as R_1 .

Step 4. R_2 is not transitive: (1, 3) and (3, 14) are in R_2 since the differences are 2 and 11, but $|1 - 14| = 13$, so $(1, 14) \notin R_2$.

Step 5. Neither relation is transitive, so neither is an equivalence relation.

Answer: (2) Neither R_1 nor R_2 is an equivalence relation

98 Answer: A

Ref 6567

Medium ~3 min Relations

List the symmetric relations on $\{x, y\}$ and check which of them are transitive.

Step 1. $\{x, y\} \times \{x, y\}$ has 4 pairs, so there are $2^4 = 16$ relations.

Step 2. For symmetry, (x, y) and (y, x) are both in or both out, while (x, x) and (y, y) are free. That gives $2^3 = 8$

symmetric relations.

Step 3. The 4 symmetric relations without (x, y) are subsets of $\{(x, x), (y, y)\}$. They are all transitive, since no chain of two different pairs exists.

Step 4. If (x, y) and (y, x) are in R , transitivity needs (x, x) from $(x, y), (y, x)$ and (y, y) from $(y, x), (x, y)$. Only the full relation works, so 1 more.

Step 5. So 5 relations are both symmetric and transitive, and the probability is $\frac{5}{16}$.

Answer: (A) $\frac{5}{16}$

COMMON TRAPS

- **Counting only transitive relations and ignoring symmetric.** Both conditions mean intersection: a relation must be symmetric and transitive simultaneously. *Avoid it:* After applying transitivity, explicitly re-check symmetry before counting a relation as favourable.
- **Treating off-diagonal pairs independently under symmetry.** In a symmetric relation, off-diagonal ordered pairs occur in mirror pairs: both present or both absent. *Avoid it:* Bundle (x, y) and (y, x) as one binary choice before testing transitivity.

REVIEW Relation as subset of Cartesian product, not as a function unless explicitly stated. · Practice checking transitivity by composing ordered pairs, especially (x, y) with (y, x) .

99 Answer: 4

Ref 6586

Medium ~2 min Functions

QUICK APPROACH Work out the set of values that each branch produces, and check whether these sets overlap or leave gaps.

Find the set of values each branch produces and check that the three sets split $\mathbb{N}$ with no overlap.

Step 1. Even n : $f(n) = 2n$ gives 4, 8, 12, ..., each multiple of 4 exactly once.

Step 2. $n = 3, 7, 11, \dots$: $f(n) = n - 1$ gives 2, 6, 10, ..., each number of the form $4k + 2$ exactly once.

Step 3. $n = 1, 5, 9, \dots$: $f(n) = \frac{n+1}{2}$ gives 1, 3, 5, 7, ..., each odd number exactly once.

Step 4. Each branch is one-one, and the three images (odd numbers, numbers $4k + 2$, multiples of 4) do not overlap. So f is one-one.

Step 5. Every natural number is odd, of the form $4k + 2$, or a multiple of 4, so each one is a value of f . So f is onto.

Answer: (4) one-one and onto

COMMON TRAPS

- **Judging onto from one branch.** A single branch such as $2n$ misses many numbers, but the other branches can supply them, so onto-ness depends on the union of all images. *Avoid it:* Write the image of every branch and check their union.

REVIEW Testing piecewise functions for one-one and onto.

100 Answer: 5264

Ref 6670

Medium ~4 min Sets and their representation

A is the set of numbers not divisible by 3 or 5, so remove the multiples of 3 and of 5 from the even numbers up to 200.

Step 1. $B = \{2, 4, \dots, 200\}$, and its sum is $2 \times \frac{100 \times 101}{2} = 10100$.

Step 2. $45 = 3^2 \times 5$, so H.C.F. $(n, 45) = 1$ exactly when 3 and 5 do not divide n .

Step 3. Even multiples of 3 are the multiples of 6 up to 198: 33 terms with sum $6 \times \frac{33 \times 34}{2} = 3366$.

Step 4. Even multiples of 5 are the multiples of 10 up to 200: sum $10 \times \frac{20 \times 21}{2} = 2100$. Multiples of 30 up to 180 are counted twice: sum $30 \times \frac{6 \times 7}{2} = 630$.

Step 5. By inclusion-exclusion, the required sum is $10100 - 3366 - 2100 + 630 = 5264$.

Answer: 5264

COMMON TRAPS

- **Counting valid elements and then summing them as the first 53 even numbers.** A subset obtained by excluding multiples is generally not an AP. Count can be found by inclusion-exclusion, but sum requires inclusion-exclusion on sums. *Avoid it:* After filtering, list the first few valid B-elements: 2, 4, 8, 14, 16, 22, ... Since differences are not constant, do not use a simple AP with $d=2$.
- **Using inclusion-exclusion for counts but not for sums.** For sums: valid sum = total sum in B – sum(multiples of 6) – sum(multiples of 10) + sum(multiples of 30). *Avoid it:* Whenever the final question says “sum of all the elements,” write a separate line for sums, not just number of terms.

REVISE Inclusion-exclusion for sums, not just counts. · Practice translating set-builder notation by listing the first few elements.

101 Answer: 1

Ref 6779

Medium ~3 min Logic

QUICK APPROACH For each wrong choice of ∇ or Δ , look for one truth assignment that breaks the implication.

Fix ∇ and Δ by testing which choices keep the implication always true, then simplify.

Step 1. If $\nabla = \wedge$, take $p = q = T$ and $r = F$. Then $p \wedge q$ is true but $(p \Delta q) \wedge r$ is false. So $\nabla = \vee$.

Step 2. With $\nabla = \vee$, if $\Delta = \wedge$, take $p = T, q = F, r = F$. Then $p \vee q$ is true but $(p \wedge q) \vee r$ is false. So $\Delta = \vee$.

Step 3. Check: $(p \vee q) \Rightarrow (p \vee q \vee r)$ is always true, so $\Delta = \nabla = \vee$ works.

Step 4. Then $(p \nabla q) \Delta r = p \vee q \vee r = (p \vee r) \vee q = (p \Delta r) \vee q$.

Step 5. At $p = T, q = F, r = F$ the statement is true, while $(p \vee r) \wedge q$ and $(p \wedge r) \vee q$ are false. So options (2), (3) and (4) do not match.

Answer: (1) $(p \Delta r) \vee q$ **COMMON TRAPS**

- **Thinking $\wedge$ makes an implication safer.** An implication fails when its premise is true and its conclusion is false, so a stronger conclusion with $\wedge$ can break it. *Avoid it:* Look for an assignment with a true premise and a false conclusion.

REVISE Tautologies and testing implications with truth values.

102 Answer: 702

Ref 6793

Medium ~4 min Sets and their representation

QUICK APPROACH Apply the concept that the smallest size of the sum set $A+A$ occurs when A forms an arithmetic progression, and use the formula for the sum of an arithmetic progression starting from the first to the last term.

A set of k integers has a sumset of at least $2k - 1$ elements, with equality only for an arithmetic progression.

Step 1. A has $1 + 18 + 1 = 20$ elements, so $A + A$ has at least $2 \times 20 - 1 = 39$ elements.

Step 2. It has exactly 39, so A must be an arithmetic progression.

Step 3. The progression runs from 1 to 77 in 19 steps, so the common difference is $\frac{77-1}{19} = 4$. The terms are 1, 5, 9, ..., 77.

Step 4. The sum of all 20 terms is $\frac{20}{2}(1 + 77) = 780$.

Step 5. Remove the end terms: $a_1 + a_2 + \dots + a_{18} = 780 - 1 - 77 = 702$.

Answer: 702

COMMON TRAPS

- **Not seeing that 39 is the smallest possible size.** A set of 20 integers has a sumset of at least 39 elements, and equality forces an arithmetic progression. *Avoid it:* Compare the given sumset size with $2k - 1$.
- **Summing all 20 terms instead of 18.** The required sum leaves out 1 and 77, so subtract them from 780. *Avoid it:* Reread which terms the question adds.

REVISE Sums of arithmetic progressions.

103 Answer: 2

Ref 6800

Medium ~3 min Functions

Step 1: The given statement is $p \Rightarrow q \vee p \Rightarrow r$.

Step 2: Using the implication equivalence, $p \Rightarrow q$ is equivalent to $\sim p \vee q$.

Step 3: Therefore, $p \Rightarrow q \vee p \Rightarrow r$ can be written as $(\sim p \vee q) \vee (\sim p \vee r)$.

Step 4: Simplifying, we get $\sim p \vee q \vee r$.

Step 5: Now, evaluate each option and find the one not equivalent:

- Option 1: $p \wedge \sim r \Rightarrow q$ is equivalent to $\sim (p \wedge \sim r) \vee q$ which simplifies to $\sim p \vee r \vee q$, equivalent to $\sim p \vee q \vee r$.
- Option 2: $\sim q \Rightarrow \sim r \vee p$ is equivalent to $q \vee \sim r \vee p$, not the same as $\sim p \vee q \vee r$.
- Option 3: $p \Rightarrow q \vee r$ is equivalent to $\sim p \vee q \vee r$, the same as the given statement.
- Option 4: $p \wedge \sim q \Rightarrow r$ is equivalent to $\sim (p \wedge \sim q) \vee r$, which

is $\sim p \vee q \vee r$.

Hence, Option 2 is not equivalent.

104 Answer: 8

Ref 6837

Medium ~5 min Relations

QUICK APPROACH Remove the pairs with $n = 0$ and $n = 1$, then count the prime powers up to 50.

$R_1 - R_2$ keeps the pairs (p, p^n) with $n \geq 2$ whose second entry is at most 50.

Step 1. Both relations contain $(p, 1)$ and (p, p) for every prime $p \leq 50$, so these pairs cancel. What remains is (p, p^n) with $n \geq 2$ and $p^n \leq 50$.

Step 2. For $p = 2$: 4, 8, 16, 32, which is 4 pairs.

Step 3. For $p = 3$: 9, 27, which is 2 pairs. For $p = 5$: 25, which is 1 pair. For $p = 7$: 49, which is 1 pair.

Step 4. For $p \geq 11$, $p^2 > 50$, so no more pairs occur.

Step 5. The total is $4 + 2 + 1 + 1 = 8$.

Answer: 8

COMMON TRAPS

- **Counting p^0 and p^1 .** The pairs with $n = 0$ or $n = 1$ lie in both relations, so the difference removes them. *Avoid it:* Start the count at $n = 2$.
- **Forgetting the bound 50.** The relations live on $\{1, \dots, 50\}$, so $64 = 2^6$ and $81 = 3^4$ do not count. *Avoid it:* Check each power against 50 before counting it.

REVISE Set difference of relations and prime powers.

105 Answer: 4

Ref 6838

Medium ~3 min Relations

Step 1: For R to be an equivalence relation, it must be reflexive, symmetric, and transitive.

Step 2: Reflexive: $3x + \alpha x$ is a multiple of 7 $\Rightarrow (3 + \alpha)x$ is a multiple of 7. This implies $3 + \alpha \equiv 0 \pmod{7} \Rightarrow \alpha \equiv 4 \pmod{7}$.

Step 3: Symmetric: If $3x + \alpha y \equiv 0 \pmod{7}$ then $3y + \alpha x \equiv 0 \pmod{7}$. If $\alpha \equiv 4 \pmod{7}$, it remains symmetric.

Step 4: Transitive: If $3x + \alpha y \equiv 0 \pmod{7}$ and $3y + \alpha z \equiv 0 \pmod{7}$, then $3x + \alpha z \equiv 0 \pmod{7}$ follows naturally.

Hence, to satisfy all three, $\alpha \equiv 4 \pmod{7}$.

Thus, Option 4 is correct.

COMMON TRAPS

- **Treating the relation as divisibility of α alone instead of the whole expression.** For reflexivity, substitute $y = x$ because (x, x) must belong to R . This gives $(3 + \alpha)x$, so the coefficient condition is $3 + \alpha \equiv 0 \pmod{7}$, not $\alpha \equiv 0 \pmod{7}$. *Avoid it:* Whenever a relation is defined by an expression in x and y , test reflexivity by explicitly replacing y with x before making any divisibility conclusion.
- **Confusing remainder 4 modulo 7 with multiple of 4.** $\alpha \equiv 4 \pmod{7}$ means $\alpha = 7k + 4$, e.g. 4, 11, 18, 25, ... It does not mean $\alpha = 4k$. *Avoid it:* Translate every congruence into the form $\alpha = \text{modulus} \times \text{integer} + \text{remainder}$ before matching options.

REVISE Equivalence relation tests: reflexive means every (x, x) , symmetric means swap ordered pairs, transitive means compose pairs. Practice congruence translation: $\alpha \equiv 4 \pmod{7}$ means "remainder 4 when divided by 7," not "multiple of 4" or "remainder 4 when divided by 10."

106 Answer: 3

Ref 6986

Medium ~2 min Logic and Boolean Algebra

The negation of an implication statement of the form $A \Rightarrow B$ is logically equivalent to $A \wedge \neg B$. For the given Boolean statement, set $A = p \vee q$ and $B = (\neg r) \vee p$. The required negation is therefore $(p \vee q) \wedge \neg((\neg r) \vee p)$.

Apply De Morgan's law to the second part: $\neg((\neg r) \vee p) = \neg(\neg r) \wedge \neg p = r \wedge (\neg p)$. Substituting yields $(p \vee q) \wedge r \wedge (\neg p)$.

Distribute over the disjunction: $[(p \wedge \neg p) \vee (q \wedge \neg p)] \wedge r$. The term $p \wedge \neg p$ is always false, so the expression simplifies to $(q \wedge \neg p) \wedge r$, which is $(\neg p) \wedge q \wedge r$.

This matches option [3]. Option [2] $(\neg p) \wedge (\neg q) \wedge r$ encodes the common mistake of negating the antecedent fully (replacing $p \vee q$ by its full negation) instead of retaining the antecedent while negating only the consequent. Thus, the correct option is [3] $(\neg p) \wedge q \wedge r$.

107 Answer: D

Ref 7002

Medium ~2 min Relations

To identify the tautology, simplify each using $A \Rightarrow B \equiv \sim A \vee B$.

For [D]: $q \Rightarrow (\sim p \vee q) \equiv \sim q \vee (\sim p \vee q) = (\sim q \vee q) \vee \sim p \equiv T \vee \sim p \equiv T$. It is always true.

Option [B] simplifies as $p \Rightarrow (\sim p \vee q) \equiv \sim p \vee (\sim p \vee q) \equiv \sim p \vee q$, false when $p = T, q = F$. This error likely arises from conflating with the valid disjunctive syllogism $p \wedge (\sim p \vee q) \Rightarrow q$, omitting the conjunction.

[A]: $(\sim p \vee q) \Rightarrow p \equiv \sim(\sim p \vee q) \vee p \equiv (p \wedge \sim q) \vee p \equiv p$, not always true.

[C]: $(\sim p \vee q) \Rightarrow q \equiv (p \wedge \sim q) \vee q$, equivalent to $p \vee q$ (by cases), not a tautology.

Thus only [D] holds in all cases.

108 Answer: 112

Ref 7019

Medium ~3 min Sets and their representation

Count all subsets of A and remove those that miss B completely.

Step 1. A has 7 elements, so it has $2^7 = 128$ subsets.

Step 2. Since $C \subseteq A$, only the common elements matter: $A \cap B = \{3, 6, 7\}$, because $9 \notin A$.

Step 3. $C \cap B = \emptyset$ means C uses none of 3, 6, 7, so C is a subset of $\{1, 2, 4, 5\}$. There are $2^4 = 16$ such sets.

Step 4. The required number is $128 - 16 = 112$.

Step 5. Check: choose any subset of $\{1, 2, 4, 5\}$ (16 ways) and a non-empty subset of $\{3, 6, 7\}$ (7 ways): $16 \times 7 = 112$.

Answer: 112

COMMON TRAPS

- **Using all of B instead of $A \cap B$.** Only 3, 6 and 7 can lie in C , since $9 \notin A$. *Avoid it:* Intersect B with A before counting.
- **Forgetting to subtract the bad subsets.** Count all 2^7 subsets and remove the 2^4 that avoid 3, 6 and 7. *Avoid it:* Use complement counting for at least one conditions.

REVISE Counting subsets with complement counting.

109 Answer: 3

Ref 7042

Medium ~2 min Functions

First simplify the bracket, then test each connective on the four rows.

Step 1. $(p \vee q) \Rightarrow q \equiv \sim(p \vee q) \vee q \equiv (\sim p \wedge \sim q) \vee q \equiv \sim p \vee q$.

Step 2. So we need $(p \wedge q) \Delta (\sim p \vee q)$ to be always true. Call the parts $A = p \wedge q$ and $B = \sim p \vee q$.

Step 3. $\Delta = \wedge$ and $\Delta = \vee$ both fail at $p = T, q = F$, where $A = F$ and $B = F$.

Step 4. $\Delta = \Leftrightarrow$ fails at $p = F, q = F$, where $A = F$ and $B = T$.

Step 5. $\Delta \Rightarrow$: $A \Rightarrow B$ is false only if $A = T$ and $B = F$. But $A = T$ means $q = T$, and then $B = T$. So it is a tautology.

Answer: (3) $\Rightarrow$

COMMON TRAPS

- **Choosing OR without checking the row $p = T, q = F$.** There both $p \wedge q$ and $\sim p \vee q$ are false, so their OR is false. *Avoid it:* Test every connective on all four rows.
- **Not simplifying $(p \vee q) \Rightarrow q$.** It reduces to $\sim p \vee q$, which makes the check much shorter. *Avoid it:* Simplify each part before testing connectives.

REVISE Truth tables of all four connectives.

110 Answer: 37

Ref 7048

Hard ~3 min Functions

QUICK APPROACH Use $f(a, b) = f(b, a)$ to turn $f(a, b) \geq a$ into $f(a, b) \geq \max(a, b)$, then make sure every value from 1 to 4 is taken.

Symmetry and $f(a, b) \geq a$ give $f(a, b) \geq \max(a, b)$, so group the unordered pairs by their larger entry.

Step 1. The pairs $\{1, 4\}, \{2, 4\}, \{3, 4\}, \{4, 4\}$ must take the value 4. The pairs $\{1, 3\}, \{2, 3\}, \{3, 3\}$ take values in $\{3, 4\}$. The pairs $\{1, 2\}, \{2, 2\}$ take values in $\{2, 3, 4\}$. The pair $\{1, 1\}$ can take any value.

Step 2. Onto needs 1 as a value, and only $\{1, 1\}$ can give it, so $f(1, 1) = 1$. The value 2 can come only from $\{1, 2\}$ or $\{2, 2\}$.

Step 3. Values for $\{1, 2\}$ and $\{2, 2\}$ that use a 2: $9 - 4 = 5$ ways. Two of these, $(2, 3)$ and $(3, 2)$, also use a 3; the other 3 do not.

Step 4. If 3 is already used, the pairs $\{1, 3\}, \{2, 3\}, \{3, 3\}$ are free: $2^3 = 8$ ways. If not, they must use a 3: $8 - 1 = 7$ ways.

Step 5. The total is $2 \times 8 + 3 \times 7 = 37$.

Answer: 37

COMMON TRAPS

- **Counting ordered pairs separately.** Since $f(a, b) = f(b, a)$, one value is chosen for each unordered pair, not two. *Avoid it:* List the 10 unordered pairs before counting.
- **Forgetting the onto condition.** Each of 1, 2, 3 and 4 must be a value, which fixes $f(1, 1) = 1$ and forces a 2 and a 3 somewhere. *Avoid it:* For each value of the codomain, find the pairs that can produce it.

REVISE Counting onto functions with restrictions.

111 Answer: 150

Ref 7073

Medium ~3 min Numbers

QUICK APPROACH Write 36 as $2^2 \times 3^2$ and turn the condition into rules about divisibility by 2, 4 and 3.

Since $36 = 2^2 \times 3^2$, a greatest common divisor of 2 means n is even, not a multiple of 4, and not a multiple of 3.

Step 1. Even but not a multiple of 4 means $n = 4k + 2$. The three-digit numbers of this form run from 102 to 998, so there are $\frac{998-102}{4} + 1 = 225$ of them.

Step 2. Remove those divisible by 3. A number that leaves remainder 2 on division by 4 and is divisible by 3 leaves remainder 6 on division by 12.

Step 3. These run from 102 to 990, so there are $\frac{990-102}{12} + 1 = 75$ of them.

Step 4. The count is $225 - 75 = 150$.

Answer: 150

COMMON TRAPS

- **Allowing multiples of 4.** If 4 divides n , the greatest common divisor with 36 is at least 4, so n must have exactly one factor 2. *Avoid it:* Count only numbers of the form $4k + 2$.
- **Forgetting the factor 3.** A multiple of 6 shares the factor 3 with 36, which makes the greatest common divisor 6. *Avoid it:* Remove the numbers divisible by 3 after the even count.

REVISE Greatest common divisor conditions using prime factors.

112 Answer: 1633

Ref 7089

Medium ~3 min Sets and their representation

QUICK APPROACH Factorise 24 and use inclusion-exclusion on sums of multiples.

$24 = 2^3 \times 3$, so $\text{HCF}(\alpha, 24) = 1$ means α is divisible by neither 2 nor 3. Subtract the multiples of 2 or 3 from the full sum.

Step 1. $1 + 2 + \dots + 100 = 5050$.

Step 2. Multiples of 2: $2(1 + 2 + \dots + 50) = 2550$. Multiples of 3: $3(1 + 2 + \dots + 33) = 1683$. Multiples of 6: $6(1 + 2 + \dots + 16) = 816$.

Step 3. By inclusion-exclusion, the multiples of 2 or 3 add up to $2550 + 1683 - 816 = 3417$.

Step 4. The required sum is $5050 - 3417 = 1633$.

Answer: 1633

COMMON TRAPS

- **Removing multiples of 6 twice.** Multiples of 6 are counted among the multiples of 2 and again among the multiples of 3, so they are added back once. *Avoid it:* Write the inclusion-exclusion formula before adding.

REVISE Inclusion-exclusion for sums of multiples.

113 Answer: 1

Ref 7177

Medium ~2 min Functions

Split on the truth value of p and see what r must be.

Step 1. If $p = F$, both $p \wedge r$ and $p \wedge \neg q$ are false, so the biconditional is true. This matches $\neg p = T$ for any r .

Step 2. If $p = T$, the statement becomes $r \Leftrightarrow \neg q$. To match $\neg p = F$, it must be false for every q .

Step 3. $r \Leftrightarrow \neg q$ is always false exactly when r always has the opposite value of $\neg q$, that is $r = q$.

Step 4. Check $r = q$: $(p \wedge q) \Leftrightarrow (p \wedge \neg q)$. When $p = T$ this is $q \Leftrightarrow \neg q = F$, and when $p = F$ it is T . So it equals $\neg p$.

Step 5. For $r = \neg q$ the statement is always true, and $r = p$ or $r = \neg p$ gives a value that depends on q .

Answer: (1) q

COMMON TRAPS

- **Choosing option B by copying the target $\neg p$ into r .** r is not the whole expression; r is only inside the subexpression " $(p \wedge r)$ ". The entire statement " $(p \wedge r) \Leftrightarrow (p \wedge (\neg q))$ " must have the same truth values as " $\neg p$ ". *Avoid it:* Substitute each candidate for r into the full expression before selecting. Do not choose an option just because it repeats the target expression.
- **Checking only the $p = \text{false}$ case from $\neg p$.** For $p = \text{false}$, both " $p \wedge r$ " and " $p \wedge \neg q$ " are false, so the biconditional is automatically true regardless of r . The deciding case is $p = \text{true}$, where the expression reduces to " $r \Leftrightarrow \neg q$ " and must be false, so r must equal q . *Avoid it:* Always check both target-true and target-false rows of a logical equivalence.

REVISE The truth tables for $\wedge$, $\neg$, and $\Leftrightarrow$, especially the fact that $\Leftrightarrow$ is false when its inputs differ. Practise simplifying logical expressions by splitting into cases $p = \text{true}$ and $p = \text{false}$.

114 Answer: C

Ref 4993

Medium ~2 min Functions

An inverse of $g \circ f : A \rightarrow C$ exists only if $g \circ f$ is one-one and onto.

Step 1. Let $h = g \circ f$. Since h^{-1} exists, h is a bijection.

Step 2. Suppose $f(x_1) = f(x_2)$. Then $g(f(x_1)) = g(f(x_2))$, so $h(x_1) = h(x_2)$. As h is one-one, $x_1 = x_2$. So f is one-one.

Step 3. Take any $c \in C$. Since h is onto, some $a \in A$ has $g(f(a)) = c$. So c is the image under g of the element $f(a)$ of B , and g is onto.

Step 4. Nothing forces f to be onto or g to be one-one. For example, f can miss an element of B on which g repeats a value.

Answer: (C) f is one-one and g is onto

COMMON TRAPS

- **Thinking both functions must be one-one.** Only f must be one-one; g needs to be one-one only on the values f takes. *Avoid it:* Find which function's failure would break $g \circ f$ being one-one.
- **Thinking both functions must be onto.** Only g must be onto C ; f may miss elements of B . *Avoid it:* Trace an element of C back through g , then f .

REVISE How one-one and onto behave under composition.

115 Answer: 1

Ref 4778

Medium ~3 min Quantifiers and Negation

The given statement is $\forall M > 0, \exists x \in S$ such that $x \geq M$. To find its negation, apply the rule that $\neg(\forall y P(y)) \equiv \exists y \neg P(y)$ and $\neg(\exists z Q(z)) \equiv \forall z \neg Q(z)$. Thus, the negation is $\exists M > 0$ such that $\neg(\exists x \in S (x \geq M))$, which simplifies to $\exists M > 0 \forall x \in S (x < M)$. This matches option [1] exactly: there exists $M > 0$, such that $x < M$ for all $x \in S$.

Option [3] encodes the common quantifier-negation error of flipping only the predicate inequality while leaving the inner existential quantifier unchanged (instead of converting $\exists$ to $\forall$). Option [4] is the negation of a different (universal) statement, and [2] is essentially the original claim. The logical

steps confirm [1] as the precise negation, equivalent to saying S is bounded above.

116 Answer: 3

Ref 4833

Medium ~3 min Functions

QUICK APPROACH Find f^{-1} and g^{-1} explicitly and clear denominators in the given equation.

Find both inverses, then solve the resulting quadratic.

Step 1. From $y = \frac{x-2}{x-3}$: $y(x-3) = x-2$, so $x(y-1) = 3y-2$. Hence $f^{-1}(x) = \frac{3x-2}{x-1}$.

Step 2. From $y = 2x-3$: $g^{-1}(x) = \frac{x+3}{2}$.

Step 3. $\frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$. Multiplying by $2(x-1)$: $6x-4+(x+3)(x-1) = 13(x-1)$.

Step 4. This simplifies to $x^2 + 8x - 7 = 13x - 13$, so $x^2 - 5x + 6 = 0$ and $x = 2$ or $x = 3$. Both lie in $\mathbb{R} - \{1\}$.

Step 5. The sum of the values is $2 + 3 = 5$.

Answer: (3) 5

COMMON TRAPS

- **Using $f(x)$ instead of its inverse.** The equation involves $f^{-1}(x) = \frac{3x-2}{x-1}$, not $f(x)$. *Avoid it:* Swap and solve for the inverse first.
- **Ignoring the domain.** The inverse is defined on $\mathbb{R} - \{1\}$, so a root at $x = 1$ would be rejected. *Avoid it:* Check each root against the domain.

REVISE Finding inverses of rational functions.

117 Answer: 1

Ref 4981

Hard ~5 min Sets and their representation

The number of relations from a set X to a set Y is $2^{|X||Y|}$, so $p = |A \cap B| \times |A \cap C|$.

Step 1. List the integer points of B (disc of radius 2 about the origin): $(0, 0)$, $(\pm 1, 0)$, $(\pm 2, 0)$, $(0, \pm 1)$, $(0, \pm 2)$, $(\pm 1, \pm 1)$. That is 13 points.

Step 2. Keep those with $(x-2)^2 + y^2 \leq 4$: $(0, 0)$, $(1, 0)$, $(2, 0)$, $(1, 1)$, $(1, -1)$. So $|A \cap B| = 5$.

Step 3. List the 13 integer points of A (disc of radius 2 about $(2, 0)$). Keep those with $(x-2)^2 + (y-2)^2 \leq 4$: $(2, 0)$, $(2, 1)$, $(2, 2)$, $(1, 1)$, $(3, 1)$. So $|A \cap C| = 5$.

Step 4. Hence $p = 5 \times 5 = 25$.

Answer: (1) 25

118 Answer: 3

Ref 5103

Medium ~3 min Sets and their representation

Let $z = x + iy$. Then S_1 is the closed disk $(x-1)^2 + y^2 \leq 2$ (center $(1, 0)$, radius $\sqrt{2}$).

S_2 simplifies to $\operatorname{Re}((1-i)z) = x+y \geq 1$, the closed half-plane above or on the line $y = 1-x$.

S_3 is $y \leq 1$.

The intersection is this disk cut by the two half-planes. The point $(1, 0)$ satisfies $(1-1)^2 + 0^2 = 0 \leq 2$, $1+0 \geq 1$, and $0 \leq 1$. The segment from $(1, 0)$ to $(1, 1)$ (i.e., $z = 1 + it$, $0 \leq t \leq 1$) lies entirely in the intersection: $x+y = 1+t \geq 1$, $y = t \leq 1$, and $t^2 \leq 1 < 2$. Thus it contains a line segment, so infinitely many elements.

Option 1 (singleton) arises from mistakenly solving the three boundary equations $(x-1)^2 + y^2 = 2$, $x+y = 1$, $y = 1$, which intersect only at $(0, 1)$ (on the circle), ignoring the inequalities that fill a region.

Hence, $S_1 \cap S_2 \cap S_3$ has infinitely many elements, option (3).

119 Answer: 3

Ref 5104

Medium ~1 min Logical Connectives

Rewrite each option using $a \rightarrow b \equiv a \vee \neg b$ and De Morgan's law.

Step 1. Option (3): $\neg(p \rightarrow q) \equiv \neg(\neg p \vee q) \equiv p \wedge \neg q$. This is the given expression.

Step 2. Option (2): $\neg(q \rightarrow p) \equiv \neg(\neg q \vee p) \equiv q \wedge \neg p$. The roles of p and q are swapped, so it differs.

Step 3. Option (1): $\neg p \rightarrow \neg q \equiv p \vee \neg q$, which is true when $p = T$, $q = T$, unlike $p \wedge \neg q$.

Step 4. Option (4): $\neg(p \rightarrow \neg q) \equiv \neg(\neg p \vee \neg q) \equiv p \wedge q$, which also differs.

Answer: (3) $\neg(p \rightarrow q)$

COMMON TRAPS

- **Swapping p and q when negating an implication.** $\neg(p \rightarrow q) \equiv p \wedge \neg q$, while $\neg(q \rightarrow p) \equiv q \wedge \neg p$. *Avoid it:* Write $a \rightarrow b \equiv \neg a \vee b$ with a and b marked first.
- **Negating only part of a disjunction.** De Morgan's law negates both parts and changes $\vee$ to $\wedge$. *Avoid it:* Check the result on one row of the truth table.

REVISE The negation of an implication.

120 Answer: 2

Ref 5203

Medium ~3 min Functions

The recurrence fixes f completely; then test each statement.

Step 1. Let $f(1) = k \geq 1$. Then $f(n) = nk$ for all n .

Step 2. If f is onto, 1 is in the range $\{k, 2k, \dots\}$, so $k = 1$ and $f(n) = n$. Statement (1) is true.

Step 3. $f(m) = f(n)$ gives $mk = nk$, so $m = n$: f is one-one and (3) is true. If $g(m) = g(n)$ then $fog(m) = fog(n)$, so fog one-one forces g one-one: (4) is true.

Step 4. Take $f(n) = n$ and $g(1) = g(2) = 1$, $g(n) = n-1$ for $n \geq 3$. Then g is onto but $fog(1) = fog(2)$, so fog is not one-one. Statement (2) is not true.

Answer: (2) If g is onto, then fog is one-one

121 Answer: C

Ref 5213

Medium ~2 min Functions

Step 1: The given statement is $(p \vee r) \Rightarrow (q \vee r)$. The implication $A \Rightarrow B$ is logically equivalent to $\sim A \vee B$.

Step 2: So, the given statement becomes $\sim (p \vee r) \vee (q \vee r)$.

Step 3: Now, the negation of $A \Rightarrow B$ is $A \wedge \sim B$.

Step 4: For the expression $A = (p \vee r)$ and $B = (q \vee r)$, the negation becomes $A \wedge \sim B = (p \vee r) \wedge \sim (q \vee r)$.

Step 5: Applying De Morgan's Laws, $\sim (q \vee r)$ becomes $\sim q \wedge \sim r$.

Step 6: So, the expression is $(p \vee r) \wedge (\sim q \wedge \sim r)$.

Step 7: Applying the distributive laws: $(p \wedge \sim q \wedge \sim r) \vee (r \wedge \sim q \wedge \sim r)$. The second term $r \wedge \sim r$ becomes false, so it reduces to $p \wedge \sim q \wedge \sim r$.

Therefore, the correct option is C: $p \wedge \sim q \wedge \sim r$.

COMMON TRAPS

- **Reversing the implication-negation rule.** The only false case of $A \Rightarrow B$ is when A is true and B is false, so $\sim (A \Rightarrow B) = A \wedge \sim B$. *Avoid it:* Before algebra, say in words: 'antecedent true, consequent false.' Mark $A = (p \vee r)$ and $B = (q \vee r)$, then write $A \wedge \sim B$.
- **Negating the consequent incorrectly by De Morgan's law.** $\sim (X \vee Y) = \sim X \wedge \sim Y$; the OR becomes AND because both parts must be false for the original OR to be false. *Avoid it:* Circle the connective inside the negated bracket and force-change it: OR under negation becomes AND.

REVIEW Memorize and verify the row-based rule: $\sim (A \Rightarrow B) = A \wedge \sim B$. Use option testing: plug each option into the original implication and check whether it makes the implication false.

122 Answer: D

Ref 5256

Medium ~2 min Union, intersection and complement

Rewrite both implications as disjunctions and combine them with the distributive law.

Step 1. $p \Rightarrow q \equiv \sim p \vee q$ and $q \Rightarrow \sim p \equiv \sim q \vee \sim p$.

Step 2. The expression is $(\sim p \vee q) \wedge (\sim q \vee \sim p)$.

Step 3. By the distributive law this is $\sim p \vee (q \wedge \sim q)$.

Step 4. Since $q \wedge \sim q$ is always false, the expression is $\sim p$.

Step 5. Check with truth values: when $p = T$, the first implication needs $q = T$, but then $q \Rightarrow \sim p$ is $T \Rightarrow F$, so the whole is false. When $p = F$, both implications are true. This matches $\sim p$.

Answer: (D) $\sim p$

COMMON TRAPS

- **Putting the negation on the consequent.** $p \Rightarrow q \equiv \sim p \vee q$; only the antecedent is negated. *Avoid it:* Test the rewrite at $p = T, q = F$, where it must be false.
- **Missing the contradiction after distributing.** $(\sim p \vee q) \wedge (\sim p \vee \sim q) \equiv \sim p \vee (q \wedge \sim q)$, and $q \wedge \sim q$ is always false. *Avoid it:* Look for a term and its negation joined by AND.

REVIEW Implication rewriting and the distributive laws.

123 Answer: 3

Ref 5425

Medium ~2 min Logic

Step 1: The given statement 'The match will be played only if the weather is good and ground is not wet' can be represented logically as: $P \rightarrow (Q \wedge \sim R)$, where P: the match will be played, Q: the weather is good, and R: the ground is wet.

Step 2: The statement ' $P \rightarrow Q$ ' logically is equivalent to ' $\sim P \vee Q$ '. Therefore, our statement becomes ' $\sim P \vee (Q \wedge \sim R)$ '.

Step 3: To find the negation, we need to negate the entire statement: $\sim(\sim P \vee (Q \wedge \sim R))$. This is equivalent to $P \wedge \sim(Q \wedge \sim R)$ using De Morgan's law.

Step 4: Applying De Morgan's laws again, $\sim(Q \wedge \sim R)$ becomes $\sim Q \vee R$. So, the negation is $P \wedge (\sim Q \vee R)$.

Step 5: Reinterpreting it back into the context of the statement, it means the match will be played and either the weather is not good or the ground is wet. Thus, option 3 is the correct negation.

COMMON TRAPS

- **Treating "only if" as "if" and reversing the implication.** "P only if Q" translates as $P \rightarrow Q$, not $Q \rightarrow P$. *Avoid it:* Replace the sentence by "If the match is played, then the weather is good and the ground is not wet."
- **Negating the consequent instead of negating the whole implication.** The negation of $P \rightarrow S$ is $P \wedge \sim S$, not $P \rightarrow \sim S$. *Avoid it:* First rewrite $P \rightarrow S$ as $\sim P \vee S$, then negate the whole expression.

REVIEW The conditional templates: P if $Q = Q \rightarrow P$; P only if $Q = P \rightarrow Q$; P iff $Q = P \leftrightarrow Q$. Practice negating conditionals using the counterexample form: $\sim(P \rightarrow Q) = P \wedge \sim Q$.

124 Answer: 80

Ref 5570

Hard ~4 min Sets and their representation

QUICK APPROACH Group the elements by remainder mod 3 and count subsets whose sum is a multiple of 3 first.

Count subsets by the remainders of their elements on division by 3.

Step 1. Remainder 0: {3, 6, 9}. Remainder 1: {1, 4}. Remainder 2: {2, 5}.

Step 2. The elements with remainder 0 never change the sum's remainder, so they give a factor of $2^3 = 8$.

Step 3. Choose i elements from {1, 4} and j from {2, 5}. The sum is a multiple of 3 when $i + 2j$ is a multiple of 3: $(i, j) = (0, 0)$ gives 1 way, $(1, 1)$ gives $2 \times 2 = 4$ ways, $(2, 2)$ gives 1 way. Total 6.

Step 4. Subsets with sum a multiple of 3: $8 \times 6 = 48$,

including the empty set.

Step 5. All $2^7 = 128$ subsets minus these 48 leaves 80. The empty set is among the 48, so all 80 are non-empty.

Answer: 80

COMMON TRAPS

- **Double handling the empty set.** The empty set has sum 0, a multiple of 3, so it is already excluded by the count of 48. *Avoid it:* Decide once where the empty set is counted.
- **Missing the (2, 2) case.** Taking both remainder-1 and both remainder-2 elements gives a sum with remainder $2 + 4 = 6$, a multiple of 3. *Avoid it:* List every (i, j) pair systematically.

REVISE Counting subsets with remainder conditions.

125 Answer: 3

Ref 5572

Medium ~2 min Sets and their representation

To negate the statement $\neg p \wedge (p \vee q)$, apply De Morgan's law for conjunction: $\neg(A \wedge B) \equiv \neg A \vee \neg B$. This yields $\neg(\neg p) \vee \neg(p \vee q)$, which is $p \vee (\neg p \wedge \neg q)$ after applying De Morgan's to the disjunction.

Distribute: $p \vee (\neg p \wedge \neg q) = (p \vee \neg p) \wedge (p \vee \neg q) \equiv \top \wedge (p \vee \neg q) \equiv p \vee \neg q$.

Alternatively, simplify the original first: $\neg p \wedge (p \vee q) = (\neg p \wedge p) \vee (\neg p \wedge q) \equiv \perp \vee (\neg p \wedge q) \equiv \neg p \wedge q$. Its negation is then $\neg(\neg p) \vee \neg q \equiv p \vee \neg q$, matching the prior result.

Option 1 ($\neg p \vee q$) encodes the specific mistake of incorrectly negating only select connectives or mishandling De Morgan's on the disjunction (a common error students may have been taught). Thus, the correct option is [3] $p \vee \neg q$.

126 Answer: 2

Ref 5624

Medium ~2 min Functions

The given Boolean expression is $(p \wedge \neg q) \Rightarrow (q \vee \neg p)$. To simplify, recall that any implication $A \Rightarrow B$ is logically equivalent to $\neg A \vee B$. Substituting $A = p \wedge \neg q$ and $B = q \vee \neg p$ yields $\neg(p \wedge \neg q) \vee (q \vee \neg p)$.

Apply De Morgan's law to the negation: $\neg(p \wedge \neg q) = \neg p \vee \neg(\neg q) = \neg p \vee q$. The full expression now becomes $(\neg p \vee q) \vee (q \vee \neg p)$.

By the idempotent and commutative laws of OR, repeated terms simplify directly to $\neg p \vee q$. This is the standard form for the implication $p \Rightarrow q$, as $p \Rightarrow q \equiv \neg p \vee q$.

A truth table verification confirms equivalence: the original expression is false only when p is true and q is false, matching exactly when $p \Rightarrow q$ is false. Option 3 ($q \Rightarrow p$) encodes the common mistake of reversing the implication direction, perhaps from incorrectly distributing negation or swapping antecedent/consequent in the contrapositive step. This yields $\neg q \vee p$, which has a different truth table (false when q true and p false).

Thus, the expression simplifies rigorously to option 2. The correct option is [2] $p \Rightarrow q$.

COMMON TRAPS

- **Reversing the direction of the implication.** The expression reduces to $\neg p \vee q$, which is $p \Rightarrow q$, not $q \Rightarrow p$. *Avoid it:* Match the negated letter in $\neg a \vee b$ to the antecedent a .
- **Negating $p \wedge \neg q$ incorrectly.** $\neg(p \wedge \neg q) \equiv \neg p \vee q$ by De Morgan's law. *Avoid it:* Negate each part and change AND to OR in one line.

REVISE De Morgan's laws and implication rewriting.

127 Answer: 2

Ref 5766

Medium ~3 min Sets and their representation

QUICK APPROACH Split into the cases $x \geq 0$, $-4 \leq x < 0$ and $x < -4$, and keep only the roots that lie in their own case.

Split the real line where $|x|$ and $|x + 4|$ change sign, and solve a quadratic on each piece.

Step 1. For $x \geq 0$: $(x - 3)(x + 4) = 6$, so $x^2 + x - 18 = 0$ and $x = \frac{-1 \pm \sqrt{73}}{2}$. Only $x = \frac{-1 + \sqrt{73}}{2} \approx 3.77$ is non-negative, so this piece gives one root.

Step 2. For $-4 \leq x < 0$: $(-x - 3)(x + 4) = 6$, so $x^2 + 7x + 18 = 0$. Its discriminant is $49 - 72 < 0$, so there is no root.

Step 3. For $x < -4$: $(-x - 3)(-x - 4) = 6$, so $(x + 3)(x + 4) = 6$ and $x^2 + 7x + 6 = 0$. This gives $x = -1$ or $x = -6$, and only $x = -6$ lies below -4 .

Step 4. The set is $\left\{\frac{-1 + \sqrt{73}}{2}, -6\right\}$, which has 2 elements.

Answer: (2) 2

COMMON TRAPS

- **Keeping roots outside their case.** A root of a case equation counts only if it lies in the interval that defined that case. *Avoid it:* Write the interval next to each case and test every root against it.
- **Reading $|x| - 3$ as $|x - 3|$.** $|x| - 3$ changes form at $x = 0$, while $|x - 3|$ changes form at $x = 3$, so the cases differ. *Avoid it:* Copy each absolute value exactly and find its break point before solving.

REVISE Equations with several absolute value terms.

128 Answer: 4

Ref 5826

Medium ~3 min Relations

The equivalence relation satisfies $(4, 3) \sim (c, d)$ iff $4d = 3c$, or equivalently, the ratios $4/3 = c/d$ hold with $c, d \in A = \{2, 3, \dots, 30\}$. Since $\gcd(3, 4) = 1$, for integer solutions we must have $c = 4k$ and $d = 3k$ for positive integer k (as $3c = 4d$ implies c is a multiple of 4 and d a multiple of 3). Constraints: $2 \leq 4k \leq 30$ gives $1 \leq k \leq 7$ ($k=1$ yields $c=4$; $k=7$ yields $c=28$; $k=8$ gives $c=32 > 30$). For these k , $d = 3k$ ranges from 3 to 21, all within $[2, 30]$. The pairs are thus $(4, 3)$, $(8, 6)$, $(12, 9)$, $(16, 12)$, $(20, 15)$, $(24, 18)$, $(28, 21)$: exactly 7. Option 8 encodes the specific mistake of including $k=0$, which yields the invalid $(0, 0) \notin A \times A$. This error arises from considering non-negative multiples without verifying set membership. No other solutions exist, as fractional k (e.g., $k=1/2$) yield non-integer c or d . Therefore, the number of ordered pairs is 7, option [4].

129 Answer: 2

Ref 5854

Medium ~4 min Relations

The given relation is defined by $x^3 - 3x^2y - xy^2 + 3y^3 = 0$. Factor the left side by grouping: $x^2(x - 3y) - y^2(x - 3y) = (x^2 - y^2)(x - 3y)$. Thus, $(x^2 - y^2)(x - 3y) = 0$, so either $x^2 = y^2$ or $x = 3y$. Since $x, y \in \mathbb{N}$, $x^2 = y^2$ implies $x = y$. Hence, $(x, y) \in R$ iff $x = y$ or $x = 3y$.

Reflexive: For any $x \in \mathbb{N}$, $x = x$, so $(x, x) \in R$. Thus, R is reflexive.

Symmetric: Take $x = 3, y = 1$. Then $3 = 3 \cdot 1$, so $(3, 1) \in R$. But $1 \neq 3$ and $1 \neq 3 \cdot 3$, so $(1, 3) \notin R$. Hence, R is not symmetric. (This is the error encoded in Option 3.)

Transitive: Take $x = 9, y = 3, z = 1$. Then $9 = 3 \cdot 3$ so $(9, 3) \in R$, and $3 = 3 \cdot 1$ so $(3, 1) \in R$. But $9 \neq 1$ and $9 \neq 3 \cdot 1$, so $(9, 1) \notin R$. Hence, R is not transitive.

Thus, R is reflexive but neither symmetric nor transitive, which is option [2].

130 Answer: D

Ref 5859

Medium ~2 min Logical Connectives

A tautology must be true in every row, so look for one false row in each option.

Step 1. Option (A) simplifies to $p \vee q$, which is false when $p = F, q = F$.

Step 2. Option (B): $p \rightarrow q \equiv \sim p \vee q$, so (B) is $(p \wedge q) \vee \sim p \vee q$. With $p = T, q = F$ every part is false.

Step 3. Option (C) needs $p \wedge q$ true, so it is false when $p = F$.

Step 4. Option (D): $(p \wedge q) \rightarrow (p \rightarrow q) \equiv (\sim p \vee \sim q) \vee (\sim p \vee q)$. This contains $\sim q \vee q$, which is always true.

Step 5. So (D) is true in every row. In words, if p and q are both true, then $p \rightarrow q$ is certainly true.

Answer: (D) $(p \wedge q) \rightarrow (p \rightarrow q)$

COMMON TRAPS

- **Missing the row** $p = T, q = F$. $(p \wedge q) \vee (p \rightarrow q)$ is false when $p = T, q = F$, so it is not a tautology. *Avoid it:* Check all four rows for every option.
- **Thinking an option that is true in most rows is a tautology.** A tautology must be true in every row; one false row rules it out. *Avoid it:* Search for a counterexample before accepting an option.

REVISE Truth tables for implication and conjunction.

131 Answer: 832

Ref 5906

Hard ~7 min Sets and their representation

Describe each set, then add the numbers of the form $3k + 1$ that are odd and at most 100.

Step 1. For A , $n^2 - n \leq 10000$. At $n = 100$, $n(n - 1) = 9900 \leq 10000$; at $n = 101$, $n(n - 1) = 10100 > 10000$. So $A = \{1, 2, \dots, 100\}$.

Step 2. With $k \in \mathbb{N} = \{1, 2, 3, \dots\}$, $B = \{4, 7, 10, 13, \dots\}$ and

C is the set of even numbers.

Step 3. $B - C$ keeps the odd members of B . Since $3k + 1$ is odd only when k is even, these are 7, 13, 19, ..., with common difference 6.

Step 4. Within A the terms are 7, 13, ..., 97. The number of terms is $\frac{97-7}{6} + 1 = 16$.

Step 5. The sum is $\frac{16}{2}(7 + 97) = 8 \times 104 = 832$.

Answer: 832

COMMON TRAPS

- **Including 1 in B .** With $k \in \mathbb{N} = \{1, 2, \dots\}$, the smallest member of B is $3 \times 1 + 1 = 4$, so 1 is not counted. *Avoid it:* Write the first few members of each set before intersecting.
- **Forgetting which members of B are odd.** $3k + 1$ is odd only when k is even, so the terms are 7, 13, 19, ... with difference 6. *Avoid it:* Check parity of $3k + 1$ for a few values of k .

REVISE Arithmetic progressions and their sums. · Set difference with number sets.

132 Answer: 3

Ref 5980

Medium ~3 min Functions

For $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, the domain of both requires $x \geq 0$ and $1-x \geq 0$, so $[0, 1]$. The sums $(f+g)(x)$ and $(f-g)(x)$ (or $(g-f)(x)$) are defined on this interval. However, $(f/g)(x) = f(x)/g(x)$ requires $g(x) \neq 0$, i.e., $\sqrt{1-x} \neq 0$ so $x \neq 1$. Similarly, $(g/f)(x)$ requires $f(x) \neq 0$, i.e., $\sqrt{x} \neq 0$ so $x \neq 0$. At $x = 0$, g/f involves division by zero; at $x = 1$, f/g does likewise. Thus, the common domain excludes both endpoints, yielding $0 < x < 1$. Option [2] ($0 \leq x < 1$) encodes the specific mistake of excluding only the point where the denominator of f/g vanishes while overlooking the zero in the denominator of g/f . The correct option is [3].

133 Answer: 490

Ref 6020

Medium ~3 min Functions

QUICK APPROACH Find $f(1)$ first, then list the products $m \times n$ that stay inside S .

Find $f(1)$, then see which values the products inside S force and which stay free.

Step 1. Put $m = n = 1$: $f(1) = f(1)^2$, and $f(1) \in S$ is non-zero, so $f(1) = 1$. Then $f(1 \cdot n) = f(1)f(n)$ holds for every n .

Step 2. The other products inside S are $2 \times 2 = 4$ and $2 \times 3 = 6$. So $f(4) = f(2)^2$ and $f(6) = f(2)f(3)$, and both must lie in S .

Step 3. No product inside S involves 5 or 7 except with 1, so $f(5)$ and $f(7)$ are free: $7 \times 7 = 49$ ways.

Step 4. $f(2)^2 \leq 7$ forces $f(2) = 1$ or 2. If $f(2) = 1$, then $f(4) = 1$ and $f(6) = f(3)$, so $f(3)$ has 7 choices. If $f(2) = 2$, then $f(4) = 4$ and $2f(3) \leq 7$ gives 3 choices for $f(3)$.

Step 5. The total is $(7 + 3) \times 49 = 490$.

Answer: 490

COMMON TRAPS

- **Forgetting the free values.** Elements such as 5 and 7 appear in no product inside S other than with 1, so each can map to any of the 7 elements. *Avoid it:* List every element and mark whether its image is forced or free.
- **Ignoring the codomain bound.** $f(4) = f(2)^2$ must itself lie in S , which rules out $f(2) \geq 3$. *Avoid it:* Check that every forced value stays inside the codomain.

REVISE Counting functions that satisfy a functional equation.**134** Answer: 3

Ref 3572

Medium ~2 min Functions

Step 1: Consider the proposition $(p \wedge q) \rightarrow (\neg q \vee r)$. For the implication $(A \rightarrow B)$ to be false, A must be true and B must be false. Step 2: For $(p \wedge q)$ to be true, both p and q must be true. Therefore, $p = T$ and $q = T$. Step 3: For $(\neg q \vee r)$ to be false, both $\neg q$ and r must be false (since a disjunction is false only if both terms are false). Since $q = T$, $\neg q = F$. For $\neg q \vee r$ to be false, r must be F . Step 4: Hence, the values of p , q , and r are T , T , F , respectively.

COMMON TRAPS

- **Forgetting the only false case of implication.** An implication $A \rightarrow B$ is false only when $A = T$ and $B = F$. Here $A = (p \wedge q)$, $B = (\neg q \vee r)$. *Avoid it:* Before solving, write: "False implication $\Rightarrow$ left T , right F " directly under the expression.
- **Treating conjunction as 'at least one true'.** $p \wedge q$ is true only if both $p = T$ and $q = T$. *Avoid it:* Translate $\wedge$ as 'AND: both required' beside the expression.

REVISE Memorize the single false row of implication: $T \rightarrow F = F$. • Keep a mini truth table for $\wedge$ and $\vee$ false/true conditions.**135** Answer: 1

Ref 3580

Medium ~2 min Propositional Logic

A tautology must be true in every row, so simplify each option and look for one false row.

Step 1. By De Morgan's law, $\sim (p \vee \sim q) \equiv \sim p \wedge q$.

Step 2. Option (1) is $(\sim p \wedge q) \rightarrow (p \vee q)$. If the left side is true, then q is true, so $p \vee q$ is true. The implication can never be $T \rightarrow F$, so it is a tautology.

Step 3. Option (2): take $p = F$, $q = F$. Then $\sim p \vee q = T$ and $p \vee q = F$, so the statement is false.

Step 4. Option (3): $\sim (p \wedge \sim q) \equiv \sim p \vee q$, the same left side as option (2), so the same row $p = q = F$ makes it false.

Step 5. Option (4) is $(\sim p \wedge q) \rightarrow (p \wedge q)$. With $p = F$, $q = T$ the left side is true and the right side is false.

Answer: (1) $\sim (p \vee \sim q) \rightarrow (p \vee q)$ **136** Answer: A

Ref 3716

Medium ~3 min Logic

QUICK APPROACH Start by constructing the truth table for the given expressions to determine which one is always true, applying the concept of tautology in propositional logic.

A tautology is true for every choice of truth values, so test each option for a false row.

Step 1. In option (A), the left side $p \wedge (p \rightarrow q)$ is true only when $p = T$ and $p \rightarrow q = T$, which forces $q = T$.

Step 2. So whenever the left side is true, q is true. The implication is never $T \rightarrow F$, and option (A) is a tautology. This is the rule of modus ponens.

Step 3. Option (B) is $q \rightarrow (p \wedge (p \rightarrow q))$, which equals $q \rightarrow (p \wedge q)$. With $p = F$, $q = T$ it becomes $T \rightarrow F = F$.

Step 4. Option (C) is $p \vee (p \wedge q) \equiv p$ and option (D) is $p \wedge (p \vee q) \equiv p$ by absorption. Both are false when $p = F$.

Answer: (A) $(p \wedge (p \rightarrow q)) \rightarrow q$ **COMMON TRAPS**

- **Treating "tautology" as merely "true for some values".** For two variables p and q , check all four rows: TT, TF, FT, FF. One false row is enough to reject an option. *Avoid it:* Before choosing, explicitly look for a counterexample row for each option.
- **Wrong truth value for implication when antecedent is false.** The implication $a \rightarrow b$ is false only when $a = T$ and $b = F$. If $a = F$, the implication is true. *Avoid it:* For every implication, first identify the antecedent and consequent, then ask: is it $T \rightarrow F$? Only then write F .

REVISE The truth table of implication: only $T \rightarrow F$ is false. • For every option, mark the main connective before solving.**137** Answer: 3

Ref 3734

Easy ~1 min Functions

Step 1: Understand the given statement 'If I reach the station in time, then I will catch the train.'

Step 2: In logic, a statement of the form 'If p then q ' can be written as ' $p \rightarrow q$ '.

Step 3: The contrapositive of a conditional statement ' $p \rightarrow q$ ' is ' $\neg q \rightarrow \neg p$ '.

Step 4: Here, let p be 'I reach the station in time' and q be 'I will catch the train'.

Step 5: The negation of p ($\neg p$) is 'I do not reach the station in time', and the negation of q ($\neg q$) is 'I will not catch the train'.

Step 6: The contrapositive statement becomes 'If I will not catch the train, then I do not reach the station in time'.

COMMON TRAPS

- **Confusing contrapositive with converse.** For "If p then q " / " $p \rightarrow q$ ", the converse is " $q \rightarrow p$ ", while the contrapositive is " $\neg q \rightarrow \neg p$ ". *Avoid it:* Write p and q explicitly, then apply the fixed template: contrapositive = not q implies not p .
- **Confusing contrapositive with inverse.** The inverse of " $p \rightarrow q$ " is " $\neg p \rightarrow \neg q$ "; the contrapositive is " $\neg q \rightarrow \neg p$ ". *Avoid it:* After negating, check whether the original consequent has become the new antecedent.

REVISE Memorize the template: contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$. • For every option, label it as converse, inverse, contrapositive, or neither.**138** Answer: 4

Ref 3748

Medium ~1.5 min Sets and their representation

Step 1: Understand the given statement which is a compound statement using 'or'. The statement is: ' $\sqrt{5}$ is an integer or 5 is an irrational number'.

Step 2: Use the rules of logical negation. The negation of 'p or q' is ' $\sim p$ and $\sim q$ '.

Step 3: Apply the negation to the statement:

- p: ' $\sqrt{5}$ is an integer'
- q: '5 is an irrational number'
- $\sim p$: ' $\sqrt{5}$ is not an integer'
- $\sim q$: '5 is not an irrational number'

Step 4: Combine using 'and': ' $\sqrt{5}$ is not an integer and 5 is not an irrational number.'

Therefore, the correct option is (4).

COMMON TRAPS

- **Applying De Morgan's law with the wrong connector.** For a compound statement $p \vee q$, its negation is $\neg p \wedge \neg q$, not $\neg p \vee \neg q$. *Avoid it:* Write $p = "$ $\sqrt{5}$ is an integer" and $q = "$ 5 is an irrational number" before looking at options, then mechanically transform $\neg(p \vee q)$ into $\neg p \wedge \neg q$.
- **Negating only one component statement.** Negating $p \vee q$ requires negating both p and q: $\neg p$ and $\neg q$. *Avoid it:* Underline both atomic claims: " $\sqrt{5}$ is an integer" and "5 is an irrational number". Put a negation sign in front of each before forming the final sentence.

REVIEW De Morgan's laws: $\neg(p \vee q) = \neg p \wedge \neg q$ and $\neg(p \wedge q) = \neg p \vee \neg q$. • Practise separating compound statements into atomic statements p and q before evaluating mathematical truth.

139 Answer: 3

Ref 3755

Medium ~3 min Logical Reasoning and Statements

QUICK APPROACH Start by constructing truth tables for both statements to check if (S_1) is always true (a tautology) and if (S_2) is always false (a fallacy).

Test each statement on the four rows of p and q. One row is enough to show a claim is wrong.

Step 1. For S_1 , take $p = T, q = T$. Then $q \vee p = T$, while $p \leftrightarrow \sim q$ is $T \leftrightarrow F = F$. The implication is $T \rightarrow F = F$, so S_1 is not a tautology.

Step 2. For S_2 , take $p = T, q = F$. Then $\sim q = T$, and $\sim p \leftrightarrow q$ is $F \leftrightarrow F = T$.

Step 3. So $\sim q \wedge (\sim p \leftrightarrow q) = T$ in this row. A fallacy must be false in every row, so S_2 is not a fallacy.

Step 4. Both claims fail.

Answer: (3) both (S_1) and (S_2) are not correct.

140 Answer: 28

Ref 3819

Medium ~2 min Sets and their representation

QUICK APPROACH Start by applying the formula for the total number of subsets of a set, which is 2 raised to the number of elements in the set, and use the given difference between subsets of A and B.

A set with k elements has 2^k subsets, so the condition becomes an equation in powers of 2.

Step 1. The condition gives $2^m = 2^n + 112$, that is $2^m - 2^n = 112$.

Step 2. Since $m > n$, factor out the smaller power:

$$2^n(2^{m-n} - 1) = 112 = 2^4 \times 7.$$

Step 3. The bracket $2^{m-n} - 1$ is odd, so all the factors of 2 go into 2^n . Hence $2^n = 16$ and $2^{m-n} - 1 = 7$.

Step 4. So $n = 4$ and $2^{m-n} = 8$, giving $m - n = 3$ and $m = 7$.

Step 5. Check: $2^7 - 2^4 = 128 - 16 = 112$. Then $m \times n = 7 \times 4 = 28$.

Answer: 28

COMMON TRAPS

- **Simplifying $2^m - 2^n$ to 2^{m-n} .** Exponents subtract only in division; for a difference, factor out the smaller power: $2^n(2^{m-n} - 1)$. *Avoid it:* Test the step with small numbers, such as $2^5 - 2^2 = 28 \neq 8$.
- **Using $2^k - 1$ as the number of subsets.** A set with k elements has 2^k subsets in all; $2^k - 1$ counts only the proper ones. *Avoid it:* Check the formula on a 2-element set, which has 4 subsets.

REVIEW The number of subsets of a finite set. • Practise factoring differences of powers of 2.

141 Answer: 2

Ref 3868

Medium ~2 min Relations

Step 1: The given statement is of the form "If p, then q" (i.e., if $n^3 - 1$ is even, then n is odd).

Step 2: The contrapositive of a statement "If p then q" is "If not q then not p".

Step 3: Here, 'p' is ' $n^3 - 1$ is even' and 'q' is 'n is odd'.

Step 4: The contrapositive becomes: "If n is not odd, then $n^3 - 1$ is not even."

Step 5: "n is not odd" implies "n is even" and " $n^3 - 1$ is not even" implies " $n^3 - 1$ is odd".

Step 6: Therefore, the contrapositive is: "For an integer n, if n is even, then $n^3 - 1$ is odd."

142 Answer: 4

Ref 3883

Medium ~2 min Sets and their representation

The overlap of two groups is bounded below by inclusion-exclusion and above by the smaller group.

Step 1. Let C and T be the percentages liking coffee and tea, with $n(C) = 73$, $n(T) = 65$ and $n(C \cap T) = x$.

Step 2. By inclusion-exclusion, $n(C \cup T) = 73 + 65 - x = 138 - x$. This cannot exceed 100, so $138 - x \leq 100$, which gives $x \geq 38$.

Step 3. The overlap cannot exceed either group, so $x \leq 65$.

Step 4. Hence $38 \leq x \leq 65$. The values 63, 38 and 54 are all possible; $x = 38$ simply means everyone likes at least one drink.

Step 5. Only 36 lies outside the range.

Answer: (4) 36

COMMON TRAPS

- **Excluding the boundary value 38.** From $138 - x \leq 100$ we get $x \geq 38$; at $x = 38$ the union is exactly 100%, which is allowed. *Avoid it:* Test each endpoint in the original condition before rejecting it.
- **Forgetting the upper bound on the overlap.** The overlap cannot exceed the smaller group, so $x \leq 65$. *Avoid it:* Always write both bounds: $a + b - 100 \leq x \leq \min(a, b)$.

REVISE Inclusion-exclusion for two sets. · Practise range questions on overlapping percentages.

143 Answer: 2

Ref 3926

Medium ~2 min Sets and their representation

Write both sets as intervals and test each option.

Step 1. $A = (-2, 2)$. From $|x - 2| \geq 3$: $x \leq -1$ or $x \geq 5$, so $B = (-\infty, -1] \cup [5, \infty)$.

Step 2. $A \cap B = (-2, -1]$, not $(-2, -1)$, so (1) is false.

Step 3. $B - A = (-\infty, -2] \cup [5, \infty) = \mathbb{R} - (-2, 5)$, so (2) is true.

Step 4. $A \cup B = (-\infty, 2) \cup [5, \infty)$, which leaves out 2, while $\mathbb{R} - (2, 5)$ contains 2; and $A - B = (-1, 2)$, not $[-1, 2)$. So (3) and (4) are false.

Answer: (2) $B - A = \mathbb{R} - (-2, 5)$

144 Answer: 1

Ref 3973

Medium ~2 min Relations

Simplify each option and look for a row that makes it false.

Step 1. In option (1), $(\sim p) \wedge (p \vee q) \equiv (\sim p \wedge p) \vee (\sim p \wedge q) \equiv \sim p \wedge q$.

Step 2. So option (1) is $(\sim p \wedge q) \rightarrow q$. Whenever the left side is true, q is true, so the implication is always true. It is a tautology.

Step 3. Option (2): $\sim(p \rightarrow q) \equiv p \wedge \sim q$, so the statement is $(\sim q \vee p) \vee (p \wedge \sim q) \equiv p \vee \sim q$, which is false for $p = F, q = T$.

Step 4. Option (3) is $p \leftrightarrow q$, false for $p = T, q = F$.

Step 5. Option (4): $(\sim q) \vee (p \wedge q) \equiv \sim q \vee p$. With $p = T, q = F$ the left side is true and q is false.

Answer: (1) $(\neg p) \wedge (p \vee q) \rightarrow q$

145 Answer: 4

Ref 4004

Medium ~5 min Sets and their representation

Count the memberships in two ways: set by set, and element by element.

Step 1. The 50 sets X_i have 10 elements each, so the total number of memberships is $50 \times 10 = 500$.

Step 2. Each element of T lies in exactly 20 of the X_i , so the same total is $20n(T)$. Hence $20n(T) = 500$ and $n(T) = 25$.

Step 3. The n sets Y_j have 5 elements each, giving $5n$ memberships.

Step 4. Each element of T lies in exactly 6 of the Y_j , so $5n = 6 \times 25 = 150$.

Step 5. Therefore $n = 30$.

Answer: (4) 30

COMMON TRAPS

- **Taking the size of the union as the sum of set sizes.** The sets overlap, so $50 \times 10 = 500$ counts memberships, not elements; it equals $20n(T)$. *Avoid it:* Ask whether the sets are disjoint before adding their sizes.
- **Mixing up the two multiplicities.** Use 20 with the sets X_i and 6 with the sets Y_j ; swapping them gives a wrong n . *Avoid it:* Label each family with its set count, set size and multiplicity.

REVISE Double counting of memberships in a family of sets.

146 Answer: 4

Ref 4033

Medium ~3 min Relations

To show a relation is not transitive, find a, b, c with (a, b) and (b, c) in it but (a, c) not in it.

Step 1. For R_1 , take $a = c = \sqrt{3 - \sqrt{2}}$ and $b = \sqrt[4]{2}$, so $a^2 = c^2 = 3 - \sqrt{2}$ and $b^2 = \sqrt{2}$.

Step 2. Then $a^2 + b^2 = 3$ and $b^2 + c^2 = 3$ are rational, so $(a, b), (b, c) \in R_1$. But $a^2 + c^2 = 6 - 2\sqrt{2}$ is irrational, so $(a, c) \notin R_1$. R_1 is not transitive.

Step 3. For R_2 , take $a = c = 1$ and $b = \sqrt[4]{2}$. Then $a^2 + b^2 = 1 + \sqrt{2}$ and $b^2 + c^2 = 1 + \sqrt{2}$ are irrational, so both pairs are in R_2 .

Step 4. But $a^2 + c^2 = 2$ is rational, so $(a, c) \notin R_2$. R_2 is not transitive either.

Answer: (4) Neither R_1 nor R_2 is transitive

147 Answer: 1

Ref 4166

Medium ~4 min Equations and Inequalities

Put $t = 3^x$, which is always positive, and split into cases at the breakpoints $t = 1$ and $t = 2$.

Step 1. The equation becomes $3^{t-1} + 2 = |t - 1| + |t - 2|$ with $t > 0$. The left side is always more than 2.

Step 2. For $t \geq 2$ the right side is $2t - 3$. At $t = 2$ the left side is 5 and the right side is 1, and the left side grows faster after that, so there is no root.

Step 3. For $1 \leq t < 2$ the right side is $(t - 1) + (2 - t) = 1$, but the left side is at least 3. No root.

Step 4. For $0 < t < 1$ the right side is $3 - 2t$, so we need $3^{t-1} = 1 - 2t$. The left side increases and the right side decreases, so they meet at most once.

Step 5. At $t = 0$ the left side $\frac{1}{3}$ is below the right side 1; at

$t = \frac{1}{2}$ the left side is positive and the right side is 0. So exactly one root lies in $(0, \frac{1}{2})$.

Step 6. Each $t > 0$ gives exactly one $x = \log_3 t$, so there is exactly one solution.

Answer: (1) singleton set

COMMON TRAPS

- **Ignoring that $t = 3^x$ is positive.** After the substitution, only $t > 0$ is allowed, so the first case is $0 < t < 1$, not $t < 1$.
Avoid it: Write the domain of t next to every substitution.
- **Opening $|t - 2|$ with the wrong sign.** For $1 \leq t < 2$, $t - 2$ is negative, so $|t - 2| = 2 - t$ and the right side equals 1.
Avoid it: Test a sample point such as $t = 1.5$ before removing the bars.

REVISE Case-splitting for equations with absolute values. · Use monotonicity to count roots without solving.

148 Answer: D

Ref 4179

Medium ~2 min Sets and their representation

QUICK APPROACH Start by finding the number of elements in sets A and B, then use the formula for the union of two sets involving their intersection.

Step 1: Determine the number of elements in sets A and B. Set A is all even numbers from 1 to 50, so $A = \{2, 4, 6, \dots, 50\}$. There are 25 elements in A.

Step 2: Set B includes multiples of 7 from 1 to 50, which are $\{7, 14, 21, 28, 35, 42, 49\}$. Thus, B has 7 elements.

Step 3: Find the intersection of A and B. This is the set of numbers that are multiples of 14 (the least common multiple of 2 and 7) from 1 to 50, which are $\{14, 28, 42\}$. The intersection has 3 elements.

Step 4: Use the formula for union: $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 25 + 7 - 3 = 29$.

Thus, the number of elements in the smallest subset of X that contains elements of both A and B is 29.

COMMON TRAPS

- **Choosing only A because option A equals $n(A)$.** A subset containing elements of both A and B must account for elements from both sets; A alone misses 7, 21, 35, 49 from B.
Avoid it: After selecting a candidate subset, test whether every required B element is included.
- **Choosing only B because B is smaller.** The smallest set containing both A and B is their union, not the smaller set.
Avoid it: Check whether the smaller set contains the larger set's elements; if not, it cannot contain both.

REVISE The difference between 'contains both sets' and 'common elements of both sets'. · Practice using $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ with finite sets.

149 Answer: 1

Ref 4280

Medium ~3 min Functions

QUICK APPROACH Start by using the Cauchy functional equation property to express $f(x)$ in terms of x and a constant, then use the given condition $f(1) = 2$ to find the constant.

Step 1: Given that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. This property defines f as a linear function, typically of the form $f(x) = cx$.

Step 2: Given $f(1) = 2$, we can deduce that if $f(x) = cx$, then $c = 2$. So, $f(n) = 2n$.

Step 3: The function $g(n)$ is defined as $g(n) = \sum_{k=1}^{n-1} f(k) = 2 \sum_{k=1}^{n-1} k$. This can be calculated as $2 \frac{(n-1)n}{2} = (n-1)n$.

Step 4: We need to find n such that $g(n) = 20$. Setting up the equation $(n-1)n = 20$, we solve for n .

Step 5: Solving $(n-1)n = 20$ gives $n(n-1) = 20$. The factors of 20 are (5, 4), so $n = 5$ and $n-1 = 4$ satisfies the equation.

Therefore, the value of n is 5.

COMMON TRAPS

- **Summing up to n instead of $n-1$.** The sum runs from $k = 1$ to $n-1$, so $g(n) = 2(1 + \dots + (n-1)) = n(n-1)$.
Avoid it: Circle both limits of a sum before using a formula.
- **Forgetting the factor $f(1) = 2$.** Additivity gives $f(k) = k f(1) = 2k$, not k .
Avoid it: Write out $f(1), f(2), f(3)$ before summing.

REVISE Additive functions on natural numbers. · Check a found n by adding the first few terms.

150 Answer: 2

Ref 4343

Medium ~3 min Relations

QUICK APPROACH Start by recalling that the domain of the inverse relation R^{-1} is the set of all second elements (y's) from pairs (x,y) in R, then use the given inequality to determine possible integer values of y.

Step 1: The inverse relation R^{-1} is obtained by swapping the elements in each pair of R. Thus, $R^{-1} = \{(y, x) : (x, y) \text{ belongs to } R\}$.

Step 2: The condition $x^2 + 3y^2 \leq 8$ defines a region in the xy-plane. We must find the valid integer values of x for fixed integer values of y.

Step 3: For $y = 0$, the condition becomes $x^2 \leq 8$, which implies $x = -2, -1, 0, 1, 2$. However, to determine the domain of R^{-1} , we must fix x and solve for y.

Step 4: For $x = 0$, the condition becomes $3y^2 \leq 8$, which implies $y^2 \leq 8/3$. The integer values of y satisfying this are $y = -1, 0, 1$.

Step 5: Thus, the domain of R^{-1} , which corresponds to these values of y, is $\{-1, 0, 1\}$.

151 Answer: B

Ref 4388

Medium ~2 min Functions

QUICK APPROACH Start by expressing $g(f(x))$ in terms of $f(x)$ and then form a quadratic equation in $f(x)$ by equating it to the given expression for $g(f(x))$.

Write $g(f(x))$ as a quadratic in the unknown value $y = f(\frac{5}{4})$.

Step 1. Since $g(u) = u^2 + u - 1$, we have $g(f(x)) = f(x)^2 + f(x) - 1$.

Step 2. Put $x = \frac{5}{4}$ in the given expression: $4 \times \frac{25}{16} - 10 \times \frac{5}{4} + 5 = \frac{25}{4} - \frac{50}{4} + \frac{20}{4} = -\frac{5}{4}$.

Step 3. So $y^2 + y - 1 = -\frac{5}{4}$, which gives $y^2 + y + \frac{1}{4} = 0$, that

$$\text{is } (y + \frac{1}{2})^2 = 0.$$

Step 4. Hence $y = -\frac{1}{2}$. As a check, both functions that satisfy the identity, $f(x) = 2x - 3$ and $f(x) = 2 - 2x$, give $-\frac{1}{2}$ at $x = \frac{5}{4}$.

Answer: (B) $-\frac{1}{2}$

COMMON TRAPS

- **Treating composition as multiplication.** $g(f(x))$ means put the entire value $f(x)$ into the input slot of g , so from " $g(x) = x^2 + x - 1$ " we get " $g(f(x)) = (f(x))^2 + f(x) - 1$ ". *Avoid it:* Before doing algebra, rewrite $g(\square) = (\square)^2 + (\square) - 1$ and then place $f(x)$ in every box.
- **Leaving the middle term as x instead of $f(x)$.** In a function definition, the x in $g(x) = x^2 + x - 1$ is a dummy input variable; both x^2 and x become expressions involving $f(x)$. *Avoid it:* Circle every x on the right side of $g(x)$ before substituting: $x^2 + x - 1$ has two input occurrences.

REWISE Function composition using placeholder notation: write $g(\square) = \square^2 + \square - 1$. Practice converting composite equations into equations in $y = f(a)$ for a fixed input a .

152 Answer: 4

Ref 4407

Medium ~3 min Functions

QUICK APPROACH Start by using the property of the floor function: express $[x+2]$ as $[x] + 2$, then rewrite the equation in terms of $[x]$.

Step 1: Identify the floor function in the equation. We have $[x]^2 + 2[x + 2] - 7 = 0$.

Step 2: Expand $[x + 2]$, which is equal to $[x] + 2$, since $[x + 2]$ is the greatest integer less than or equal to $x + 2$.

Step 3: Substitute and simplify the equation:

$$\begin{aligned} [x]^2 + 2([x] + 2) - 7 &= 0 \\ \Rightarrow [x]^2 + 2[x] + 4 - 7 &= 0 \\ \Rightarrow [x]^2 + 2[x] - 3 &= 0 \\ \Rightarrow ([x] - 1)([x] + 3) &= 0. \end{aligned}$$

Step 4: Solving the factors, we get $[x] = 1$ or $[x] = -3$.

Step 5: Determine the possible values of x where $[x]$ is an integer:

$$x \in [1, 2) \cup [-3, -2).$$

Step 6: Conclusion: Since x can take any real number values within these intervals, there are infinitely many solutions.

COMMON TRAPS

- **Treating $[x + 2]$ as $[x] + [2]$ incorrectly or dropping the 2.** Since 2 is an integer, $[x + 2] = [x] + 2$; the equation becomes $[x]^2 + 2[x] - 3 = 0$. *Avoid it:* Pull out only integer shifts from inside the bracket.
- **Counting only the integer values of x .** $[x] = 1$ holds for every x in $[1, 2)$ and $[x] = -3$ for every x in $[-3, -2)$, so there are infinitely many solutions. *Avoid it:* After solving for $[x]$, write the full interval for x .

REWISE Properties of the greatest integer function.

153 Answer: A

Ref 4450

Medium ~3 min Inverses of Functions

Step 1: Let $y = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$. We need to find x in terms of y .

Step 2: Rearrange for $\frac{1+y}{1-y} = \frac{8^{2x}}{8^{-2x}} = 8^{4x}$.

Step 3: Taking logarithms on both sides, $4x = \log_8 \left(\frac{1+y}{1-y} \right)$.

Step 4: Solving for x , we get $x = \frac{1}{4} \log_8 \left(\frac{1+y}{1-y} \right)$.

Step 5: Thus, $f^{-1}(x) = \frac{1}{4} \log_8 \left(\frac{1+x}{1-x} \right)$.

COMMON TRAPS

- **Dropping the negative exponent contribution when forming the ratio.** For same base, $a^m/a^n = a^{m-n}$. Here $m = 2x$, $n = -2x$, so $m - n = 2x + 2x = 4x$. *Avoid it:* Write the exponent subtraction explicitly: $2x - (-2x)$ before simplifying.
- **Choosing the reciprocal log argument.** If $y(A + B) = A - B$, then $A(1 - y) = B(1 + y)$, so $A/B = (1 + y)/(1 - y)$. *Avoid it:* Use placeholders $A = 8^{2x}$, $B = 8^{-2x}$ and collect A-terms and B-terms separately.

REWISE Exponent laws involving negative exponents: $a^m/a^n = a^{m-n}$. Practice solving $y = (A - B)/(A + B)$ for A/B using placeholders before substituting actual exponentials.

154 Answer: 3

Ref 4538

Medium ~2 min Logic and Boolean Algebra

QUICK APPROACH Start by applying De Morgan's laws to negate the entire expression, converting the negation of a disjunction into a conjunction of negations.

Step 1: The given expression is $p \vee (\sim p \wedge q)$. According to De Morgan's laws, the negation of a conjunction is the disjunction of the negations. Similarly, the negation of a disjunction is the conjunction of the negations.

Step 2: We start by finding the negation of $p \vee (\sim p \wedge q)$:

- Apply De Morgan's to $\sim (p \vee X)$: $\sim p \wedge \sim X$
- In this context, $X = \sim p \wedge q$, so:
- $\sim (p \vee (\sim p \wedge q)) = (\sim p \wedge \sim (\sim p \wedge q))$

Step 3: Now simplify $\sim (\sim p \wedge q)$:

- $\sim (\sim p \wedge q) = \sim (\sim p) \vee \sim q$ (again applying De Morgan's)
- This becomes $p \vee \sim q$

Step 4: So, $\sim (p \vee (\sim p \wedge q)) = \sim p \wedge (p \vee \sim q)$

Step 5: However, $\sim p \wedge p$ will always be false, and hence this simplifies further to $\sim p \wedge \sim q$.

Thus, the negation of the Boolean expression is $\sim p \wedge \sim q$.

155 Answer: 1

Ref 4606

Easy ~1 min Logical Connectives and Truth Tables

Step 1: For the statement $p \rightarrow (p \wedge \sim q)$ to be false, we need p to be true and $(p \wedge \sim q)$ to be false.

Step 2: If p is true, then $p \wedge \sim q$ can only be false if q is true because $\sim q$ (not q) must be false for the conjunction to be false.

Step 3: Therefore, both p and q must be true for the implication to be false. Hence, the truth values of p and q are T and T respectively.

The correct answer is (1) TT.

COMMON TRAPS

- **Using the wrong false-condition for implication.** For $A \rightarrow B$, the only false case is $A = T$ and $B = F$. Here $A = p$ and $B = (p \wedge \sim q)$, so Step 1 correctly says: " p to be true and $(p \wedge \sim q)$ to be false." *Avoid it:* Before solving, write the mini truth rule: $T \rightarrow F = F$; all other implication cases are true.
- **Treating $p \wedge \sim q$ as false by making p false, despite Step 1 fixing p true.** Because Step 1 requires $p = T$, the conjunction $p \wedge \sim q$ becomes $T \wedge \sim q$. For this to be false, $\sim q$ must be false, not p . *Avoid it:* Substitute $p = T$ into the consequent immediately: $(p \wedge \sim q) = (T \wedge \sim q)$.

REVISE Memorize the single false case of implication: $T \rightarrow F = F$. When a variable appears multiple times, assign it once and substitute consistently everywhere.

156 Answer: 1

Ref 4610

Medium ~2 min Relations

Rewrite each implication as a disjunction and use the distributive law.

Step 1. $p \rightarrow q \equiv \sim p \vee q$ and $q \rightarrow \sim p \equiv \sim q \vee \sim p$.

Step 2. The expression is $(\sim p \vee q) \wedge (\sim p \vee \sim q)$.

Step 3. By the distributive law, $(A \vee B) \wedge (A \vee C) \equiv A \vee (B \wedge C)$. With $A = \sim p$ this gives $\sim p \vee (q \wedge \sim q)$.

Step 4. Since $q \wedge \sim q$ is always false, the expression reduces to $\sim p$.

Step 5. Check: if $p = T$, the first part forces $q = T$, and then $q \rightarrow \sim p$ is $T \rightarrow F$, so the whole is false. If $p = F$, both parts are true.

Answer: (1) $\sim p$

COMMON TRAPS

- **Simplifying a conjunction of disjunctions by guesswork.** Use $(A \vee B) \wedge (A \vee C) \equiv A \vee (B \wedge C)$; here it gives $\sim p \vee (q \wedge \sim q) \equiv \sim p$. *Avoid it:* Find the common term before applying the distributive law.
- **Rewriting $q \rightarrow \sim p$ as $\sim q \vee \sim p$.** Only the antecedent is negated: $q \rightarrow \sim p \equiv \sim q \vee \sim p$. *Avoid it:* Check a rewrite on the row $T \rightarrow F$.

REVISE The distributive laws of logic. Practise converting implications into disjunctions.

157 Answer: 4

Ref 4736

Medium ~2 min Sets and their representation

Count the memberships in two ways: set by set, and element by element.

Step 1. The 50 sets X_i have 10 elements each, so the total number of memberships is $50 \times 10 = 500$.

Step 2. Each element of T lies in exactly 20 of the X_i , so the same total is $20n(T)$. Hence $20n(T) = 500$ and $n(T) = 25$.

Step 3. The n sets Y_i have 5 elements each, giving $5n$ memberships.

Step 4. Each element of T lies in exactly 6 of the Y_i , so $5n = 6 \times 25 = 150$.

Step 5. Therefore $n = 30$.

Answer: (4) 30

COMMON TRAPS

- **Taking the size of the union as the sum of set sizes.** The sets overlap, so $50 \times 10 = 500$ counts memberships, not elements; it equals $20n(T)$. *Avoid it:* Ask whether the sets are disjoint before adding their sizes.
- **Mixing up the two multiplicities.** Use 20 with the sets X_i and 6 with the sets Y_i ; swapping them gives a wrong n . *Avoid it:* Label each family with its set count, set size and multiplicity.

REVISE Double counting of memberships in a family of sets.

158 Answer: 4

Ref 4742

Medium ~2 min Sets and their representation

The overlap of two groups is bounded below by inclusion-exclusion and above by the smaller group.

Step 1. Let $n(A) = 63$, $n(B) = 76$ (in percent) and $n(A \cap B) = x$.

Step 2. By inclusion-exclusion, $n(A \cup B) = 63 + 76 - x = 139 - x$.

Step 3. The union cannot exceed 100, so $139 - x \leq 100$, which gives $x \geq 39$.

Step 4. The overlap cannot exceed the smaller group, so $x \leq 63$.

Step 5. Hence $39 \leq x \leq 63$. Of the options 65, 37, 29 and 55, only 55 lies in this range.

Answer: (4) 55

COMMON TRAPS

- **Assuming all people read at least one of A or B.** Here $n(A \cup B)$ can be at most 100%, not necessarily equal to 100%. So $63 + 76 - x \leq 100$, hence $x \geq 39$. *Avoid it:* Replace equality with an inequality unless the question explicitly says every person reads at least one of the newspapers.
- **Reversing the lower-bound inequality for x .** Since $63 + 76 - x \leq 100$, subtracting 139 gives $-x \leq -39$, and multiplying by -1 reverses the sign: $x \geq 39$. *Avoid it:* When multiplying or dividing an inequality by -1 , circle the inequality sign and reverse it.

REVISE Memorize and apply the feasible-intersection range for percentages: $\max(0, a+b-100) \leq x \leq \min(a,b)$. For survey questions, explicitly decide whether the union is equal to 100% or only at most 100%.

159 Answer: 3

Ref 2609

Medium ~5 min Sets and their representation

QUICK APPROACH Find how many students opted for at least one course using inclusion-exclusion, then take the complement.

Count the students in at least one course with inclusion-exclusion, then subtract from 140.

Step 1. Multiples up to 140: of 2, $\lfloor 140/2 \rfloor = 70$; of 3, 46; of 5, 28.

Step 2. Multiples of 6: 23; of 10: 14; of 15: 9; of 30: 4.

Step 3. Students in at least one course: $70 + 46 + 28 - 23 - 14 - 9 + 4 = 102$.

Step 4. Students in no course: $140 - 102 = 38$.

Step 5. Check: in each block of 30 numbers, 8 are coprime to 30. Four blocks give 32, and 121 to 140 adds 121, 127, 131, 133, 137, 139, which is 6 more. $32 + 6 = 38$.

Answer: (3) 38

COMMON TRAPS

- **Stopping at the union.** 102 is the number who opted for at least one course; the question asks for those who opted for none. *Avoid it:* Reread the question after the inclusion-exclusion step.
- **Forgetting to subtract the overlaps.** A number like 30 is a multiple of 2, 3 and 5, so plain addition counts it three times. *Avoid it:* Write all seven terms of inclusion-exclusion before adding.

REVISE Inclusion-exclusion for three sets with complements.

160 Answer: 2

Ref 2128

Medium ~3 min Relations

Step 1: Given that q is false and $p \wedge q \leftrightarrow r$ is true. This means $p \wedge q$ must have the same truth value as r .

Step 2: Since q is false, $p \wedge q$ is false regardless of the value of p . Therefore, r must also be false for the bi-conditional $p \wedge q \leftrightarrow r$ to be true.

Step 3: Evaluate the options as tautologies:

- Option (1): $(p \vee r) \rightarrow (p \wedge r)$ is not a tautology because if p is true and r is false, $p \vee r$ is true while $p \wedge r$ is false, making the implication false.
- Option (2): $(p \wedge r) \rightarrow (p \vee r)$ is a tautology. The implication is always true because $p \wedge r$ can only be true if both p and r are true, in which case $p \vee r$ would also be true, making the implication true.
- Option (3): $p \wedge r$ is not always true, so it's not a tautology.
- Option (4): $p \vee r$ can be false, so it's also not a tautology.

Conclusion: The only tautology is option (2).

161 Answer: 1

Ref 2166

Medium ~3 min Functions

QUICK APPROACH Rearrange $y = \frac{x}{1+x^2}$ into a quadratic in x and use the discriminant.

Treat $y = f(x)$ as a quadratic in x and require real roots.

Step 1. $y = \frac{x}{1+x^2}$ gives $yx^2 - x + y = 0$.

Step 2. If $y = 0$, then $x = 0$ works, so 0 is in the range.

Step 3. If $y \neq 0$, real x needs the discriminant $1 - 4y^2 \geq 0$, so $-\frac{1}{2} \leq y \leq \frac{1}{2}$.

Step 4. The endpoints are reached: $f(1) = \frac{1}{2}$ and $f(-1) = -\frac{1}{2}$. Every value in between has a real x by Step 3.

Step 5. So the range is $[-\frac{1}{2}, \frac{1}{2}]$. This also follows from $1 + x^2 \geq 2|x|$.

Answer: (1) $[-\frac{1}{2}, \frac{1}{2}]$

COMMON TRAPS

- **Excluding zero from the range.** $f(0) = 0$, since the denominator $1 + 0$ is not zero. *Avoid it:* Always evaluate the function at simple points like $x = 0$.
- **Using a loose bound instead of the exact one.** $|f(x)| < 1$ is true but not sharp; $1 + x^2 \geq 2|x|$ gives the exact bound $\frac{1}{2}$. *Avoid it:* Find where equality holds to get the true maximum and minimum.

REVISE The discriminant method for the range of rational functions.

162 Answer: 4

Ref 2220

Easy ~1 min Relations

Step 1: Understand the concept of contrapositive. The contrapositive of a statement of the form 'If P, then Q' is 'If not Q, then not P'.

Step 2: Apply the concept to the given statement. The given statement is 'If you are born in India, then you are a citizen of India'.

Step 3: Identify P and Q. Here, P is 'you are born in India' and Q is 'you are a citizen of India'.

Step 4: Formulate the contrapositive. The contrapositive will be 'If you are not a citizen of India, then you are not born in India'.

Thus, the correct answer is option (4).

163 Answer: 1

Ref 2258

Medium ~2 min Relations, Functions

QUICK APPROACH Work out $g(n)$ separately for odd and even n , and note whether the result is odd or even.

Find $f \circ g$ on odd and even inputs, then test one-one and onto.

Step 1. $g(n) = n - (-1)^n$, so $g(n) = n + 1$ for odd n and $g(n) = n - 1$ for even n .

Step 2. For odd n , $g(n) = n + 1$ is even, so $f(g(n)) = \frac{n+1}{2}$.

Step 3. For even n , $g(n) = n - 1$ is odd, so $f(g(n)) = \frac{(n-1)+1}{2} = \frac{n}{2}$.

Step 4. One-one: $(f \circ g)(1) = 1$ and $(f \circ g)(2) = 1$, so it is not one-one.

Step 5. Onto: for any $m \in \mathbb{N}$, $(f \circ g)(2m) = m$. So it is onto. In fact every odd n and the next number $n + 1$ map to the same value.

Answer: (1) onto but not one-one

164 Answer: 3

Ref 2263

Medium ~2 min Functions

QUICK APPROACH Substitute $t = x^2$ and find the values of $\frac{t}{1-t}$ for $0 \leq t < 1$ and for $t > 1$ separately.

A surjective map has its codomain equal to its range, so find the range of f .

Step 1. Let $t = x^2$, where $t \geq 0$ and $t \neq 1$. Then $y = \frac{t}{1-t}$.

Step 2. For $0 \leq t < 1$, y starts at 0 and increases without bound as t approaches 1. This gives $[0, \infty)$.

Step 3. For $t > 1$, write $y = -1 + \frac{1}{1-t}$. Here $\frac{1}{1-t}$ is negative, so $y < -1$. As t grows, y approaches -1 from below, and near $t = 1$ it goes to $-\infty$. This gives $(-\infty, -1)$.

Step 4. The value $y = -1$ would need $t = t - 1$, which is impossible.

Step 5. The range is $(-\infty, -1) \cup [0, \infty) = \mathbb{R} \setminus [-1, 0)$, so A must equal this set.

Answer: (3) $\mathbb{R} \setminus [-1, 0)$

165 Answer: A

Ref 2285

Medium ~3 min Logic

QUICK APPROACH For each implication, try to make the first part true and the second part false.

An implication fails only when its first part is true and its second part is false. Look for such a case.

Step 1. Option (A): take p false and q true. Then $p \vee q$ is true, but $p \vee \sim q$ is false. The implication is false, so (A) is not a tautology.

Step 2. Option (B): $p \wedge q$ true forces p and q true. Then $\sim p \vee q$ is true, so (B) never fails.

Step 3. Option (C): if p is true, then $p \vee q$ is true. If p is false, the implication is true anyway. So (C) is a tautology.

Step 4. Option (D): $p \wedge q$ true forces p true, so (D) is a tautology.

Answer: (A) $p \vee q \rightarrow p \vee (\sim q)$

166 Answer: 4

Ref 2295

Medium ~3 min Functions

QUICK APPROACH Work out $f(S)$, $g(S)$, $g(f(S))$ and $f(g(S))$ as intervals, then test each statement.

Compute each set: $g(A)$ is the set of x whose square lies in A .

Step 1. $f(S) = \{x^2 : 0 \leq x \leq 4\} = [0, 16]$.

Step 2. $g(S) = \{x : x^2 \in [0, 4]\} = [-2, 2]$, and $g(f(S)) = \{x : x^2 \in [0, 16]\} = [-4, 4]$.

Step 3. $f(g(S)) = \{x^2 : -2 \leq x \leq 2\} = [0, 4] = S$.

Step 4. Check each statement. (1) $[-4, 4] \neq [0, 4]$: true. (2) $[0, 4] \neq [0, 16]$: true. (3) $[0, 4] = S$: true. (4) $[-4, 4] = [-2, 2]$: false.

Step 5. The statement that is not true is (4).

Answer: (4) $g(f(S)) = g(S)$

COMMON TRAPS

- **Treating g as the inverse function.** $g(A)$ collects both x and $-x$, so $g(S) = [-2, 2]$, not $[0, 2]$. *Avoid it:* Include negative x whose square lies in the set.
- **Choosing a true statement.** The question asks for the statement that is not true. *Avoid it:* Mark each statement true or false before choosing.

REWISE Images and pre-images of sets under functions.

167 Answer: 1

Ref 2299

Medium ~2 min Compositions of Functions

QUICK APPROACH Apply f_2 and f_1 explicitly to get an equation for $J\left(\frac{1}{x}\right)$, then substitute $t = \frac{1}{x}$.

Peel off the outer functions one at a time.

Step 1. $(f_2 \circ J \circ f_1)(x) = f_2\left(J\left(\frac{1}{x}\right)\right) = 1 - J\left(\frac{1}{x}\right)$.

Step 2. Setting this equal to $f_3(x)$: $1 - J\left(\frac{1}{x}\right) = \frac{1}{1-x}$, so $J\left(\frac{1}{x}\right) = 1 - \frac{1}{1-x} = \frac{x}{x-1}$.

Step 3. Put $t = \frac{1}{x}$, so $x = \frac{1}{t}$. Then $J(t) = \frac{1/t}{1/t-1} = \frac{1}{1-t}$.

Step 4. So $J(x) = \frac{1}{1-x} = f_3(x)$.

Step 5. Check with $x = 2$: $f_1(2) = \frac{1}{2}$, $J\left(\frac{1}{2}\right) = 2$ and $f_2(2) = -1$, which equals $f_3(2) = \frac{1}{1-2} = -1$.

Answer: (1) $f_3(x)$

COMMON TRAPS

- **Composing in the wrong order.** In $f_2 \circ J \circ f_1$, f_1 acts first and f_2 acts last. *Avoid it:* Read compositions from right to left.
- **Forgetting to substitute back.** The equation gives $J\left(\frac{1}{x}\right)$, not $J(x)$. *Avoid it:* Replace $\frac{1}{x}$ by a new variable before reading off J .

REWISE Composition of functions and solving for an unknown function.

168 Answer: D

Ref 2356

Medium ~2 min Boolean Algebra

QUICK APPROACH Regroup each expression and look for a pair like $p \vee \sim p$.

A disjunction that contains both p and $\sim p$ is always true.

Step 1. Option (D): $(p \vee q) \vee (\sim p \vee \sim q)$ can be regrouped as $(p \vee \sim p) \vee (q \vee \sim q)$. Each bracket is always true, so (D) is a tautology.

Step 2. Option (A): with p and q both true, $\sim p \vee \sim q$ is false, so (A) is false there.

Step 3. Option (B): $(p \wedge q) \vee (p \wedge \sim q) = p \wedge (q \vee \sim q) = p$, which is false when p is false.

Step 4. Option (C): $(p \vee q) \wedge (p \vee \sim q) = p \vee (q \wedge \sim q) = p$, which is also false when p is false.

Step 5. Only (D) is true for every choice of p and q .

Answer: (D) $(p \vee q) \vee (\sim p \vee \sim q)$

169 Answer: C

Ref 2359

Medium ~3 min Power set

QUICK APPROACH Use the concept of counting subsets with at least one even number by subtracting subsets containing only odd numbers from the total subsets.

Count the complement: a product is odd only when every element is odd.

Step 1. S has 50 odd and 50 even numbers.

Step 2. The product of the elements of A is even exactly when A contains at least one even number.

Step 3. Non-empty subsets of S : $2^{100} - 1$.

Step 4. Non-empty subsets made only of odd numbers: $2^{50} - 1$. These have an odd product.

Step 5. Subsets with an even product: $(2^{100} - 1) - (2^{50} - 1) = 2^{100} - 2^{50} = 2^{50}(2^{50} - 1)$. There are 2^{100} subsets in all, including the empty set.

Answer: (C) $2^{50}(2^{50} - 1)$

170 Answer: 3

Ref 2439

Medium ~3 min Sets and their representation

QUICK APPROACH Draw a two-set Venn diagram and find the size of each separate region first.

Split the readers into three groups that do not overlap, then apply each percentage to its own group.

Step 1. Take the population as 100. Both A and B : 8. A only: $25 - 8 = 17$. B only: $20 - 8 = 12$.

Step 2. From A only, 30% look into advertisements: $0.3 \times 17 = 5.1$.

Step 3. From B only, 40% do: $0.4 \times 12 = 4.8$.

Step 4. From both, 50% do: $0.5 \times 8 = 4$.

Step 5. Total: $5.1 + 4.8 + 4 = 13.9$ out of 100, that is 13.9%. The three groups hold $17 + 12 + 8 = 37$ readers.

Answer: (3) 13.9

COMMON TRAPS

- **Applying 30% to all readers of A.** The 30% applies only to those who read A but not B , which is $25 - 8 = 17$, not 25. *Avoid it:* Find the 'only' regions before applying any percentage.
- **Counting the overlap twice.** The groups A only, B only and both are separate, so each person is counted once. *Avoid it:* Check that the three regions add up to $n(A \cup B) = 37$.

REVISE Venn diagram regions for two sets.

171 Answer: 4

Ref 2463

Hard ~3 min Composite Functions

QUICK APPROACH Write $\phi(x)$ in terms of $\tan x$ and compare it with the formula for $\tan(A - B)$.

Apply the functions from the inside out: first g , then f , then h .

Step 1. $\phi(x) = h(f(g(x))) = h(\sqrt{\tan x})$. Here g acts first, then f , then h .

Step 2. $h(u) = \frac{1-u^2}{1+u^2}$ with $u^2 = \tan x$, so $\phi(x) = \frac{1-\tan x}{1+\tan x}$. The square root and the square cancel because $\tan x > 0$ near $\frac{\pi}{3}$.

Step 3. This is $\tan(\frac{\pi}{4} - x)$, since $\tan(\frac{\pi}{4} - x) = \frac{1-\tan x}{1+\tan x}$.

Step 4. At $x = \frac{\pi}{3}$: $\phi(\frac{\pi}{3}) = \tan(\frac{\pi}{4} - \frac{\pi}{3}) = \tan(-\frac{\pi}{12})$.

Step 5. Tangent has period π , so $\tan(-\frac{\pi}{12}) = \tan(\pi - \frac{\pi}{12}) = \tan \frac{11\pi}{12}$. The value $\frac{1-\sqrt{3}}{1+\sqrt{3}}$ is negative, which matches an angle in the second quadrant. Here $\tan \frac{\pi}{3} = \sqrt{3}$, so the value is $\frac{1-\sqrt{3}}{1+\sqrt{3}} = \sqrt{3} - 2$, while $\tan \frac{\pi}{12} = 2 - \sqrt{3}$.

Answer: (4) $\tan \frac{11\pi}{12}$

COMMON TRAPS

- **Ignoring the sign of the value.** $\frac{1-\sqrt{3}}{1+\sqrt{3}}$ is negative, so it cannot equal $\tan \frac{5\pi}{12}$ or $\tan \frac{\pi}{12}$, which are positive. *Avoid it:* Check the sign of the value before matching it with an angle.
- **Applying the composition in the wrong order.** $(h \circ f) \circ g(x) = h(f(g(x)))$, so g acts first. *Avoid it:* Rewrite a composition as nested brackets before substituting.

REVISE $\tan(A \pm B)$ and the period of tangent.

172 Answer: 1

Ref 2547

Medium ~3 min Logical Connectives

QUICK APPROACH Decide whether each of P , Q and R is true or false before looking at the options.

Find the truth value of each simple statement, then evaluate each option.

Step 1. P is true, since 5 is prime. $192 = 2^6 \times 3$ has no factor 7, so Q is false. R is true, since $\text{LCM}(5, 7) = 35$.

Step 2. Option (1): $P \vee (\neg Q \wedge R) = T \vee (T \wedge T) = T$.

Step 3. Option (2): $(P \wedge Q) \vee (\neg R) = F \vee F = F$.

Step 4. Option (3): $(\neg P) \vee (Q \wedge R) = F \vee F = F$.

Step 5. Option (4): $(\neg P) \wedge (\neg Q \wedge R) = F \wedge T = F$.

Step 6. Only option (1) is true.

Answer: (1) $P \vee (\neg Q \wedge R)$

173 Answer: C

Ref 2674

Medium ~4 min Functions

QUICK APPROACH Look for two inputs on either side of $x = 1$ with the same value of $|1 - \frac{1}{x}|$, and check whether the range matches $(0, \infty)$.

Test one-one and onto separately, remembering the codomain is $(0, \infty)$.

Step 1. Onto: the codomain is $(0, \infty)$, but $f(1) = |1 - 1| = 0$. The range of f is $[0, \infty)$, which is not equal to the codomain, so f is not surjective.

Step 2. One-one: $f(2) = |1 - \frac{1}{2}| = \frac{1}{2}$ and $f(\frac{2}{3}) = |1 - \frac{3}{2}| = \frac{1}{2}$. Two different inputs give the same output, so f is not injective.

Step 3. Hence f is neither injective nor surjective.

Answer: (C) neither injective nor surjective

COMMON TRAPS

- **Ignoring the absolute value.** Without the modulus, $1 - \frac{1}{x}$ is one-one, but the modulus folds values below 1 onto values above 1. *Avoid it:* Test inputs on both sides of $x = 1$.
- **Calling f onto.** Every positive value is reached, but the range also contains 0 and differs from the codomain. *Avoid it:* Compare the range with the stated codomain exactly.

REVISE One-one and onto tests for modulus functions.

174 Answer: 1

Ref 2880

Medium ~2 min Injective and Surjective Functions

Step 1: Define the set A . The set A is the set of all real numbers except positive integers. Thus, $A = \mathbb{R} - \{1, 2, 3, \dots\}$.

Step 2: Analyze the function $f(x) = \frac{2x}{x-1}$. The function is not defined for $x = 1$, which is consistent with $x \notin A$.

Step 3: Check injectivity. Assume $f(a) = f(b)$ for $a, b \in A$. Therefore, $\frac{2a}{a-1} = \frac{2b}{b-1}$. Simplifying this gives $2a(b-1) = 2b(a-1)$, which simplifies to $ab - a = ab - b$, leading to $a = b$. Thus, f is injective.

Step 4: Check surjectivity. For $f(x)$ to be surjective, for every $y \in \mathbb{R}$, there must be an $x \in A$ such that $f(x) = y$. Solving $y = \frac{2x}{x-1}$ gives $x = \frac{y}{y-2}$. This x is not always in A because it can become a positive integer, hence f is not surjective.

Therefore, the function is injective but not surjective.

175 Answer: 3

Ref 2905

Medium ~5 min Functions: Operations

QUICK APPROACH Start by applying the chain rule to differentiate the composite function $h(x) = f(g(x))$ and express $h'(x)$ in terms of $g'(x)$ and $f'(g(x))$.

Differentiate with the chain rule and find where $h'(x) \geq 0$.

Step 1. $h(x) = f(g(x)) = e^{g(x)} - g(x)$, so $h'(x) = g'(x)(e^{g(x)} - 1)$, with $g'(x) = 2x - 1$.

Step 2. $e^{g(x)} - 1$ has the sign of $g(x) = x(x-1)$: positive for $x < 0$ or $x > 1$, negative for $0 < x < 1$.

Step 3. $g'(x)$ is negative for $x < \frac{1}{2}$ and positive for $x > \frac{1}{2}$.

Step 4. Signs of h' : on $(-\infty, 0)$, negative; on $(0, \frac{1}{2})$, positive; on $(\frac{1}{2}, 1)$, negative; on $(1, \infty)$, positive.

Step 5. So h is increasing on $[0, \frac{1}{2}] \cup [1, \infty)$. So h has a local maximum at $x = \frac{1}{2}$ and a local minimum at $x = 1$.

Answer: (3) $[0, \frac{1}{2}] \cup [1, \infty)$

176 Answer: 3

Ref 2973

Medium ~1 min Relations

QUICK APPROACH Swap the two parts of the conditional and negate both.

The contrapositive of "if p then q " is "if not q then not p ".

Step 1. Let p be "two numbers are not equal" and q be "their squares are not equal". The statement is $p \rightarrow q$.

Step 2. The contrapositive is $\sim q \rightarrow \sim p$.

Step 3. $\sim q$ is "the squares of the two numbers are equal" and $\sim p$ is "the two numbers are equal".

Step 4. So the contrapositive is "If the squares of two numbers are equal, then the numbers are equal".

Step 5. A statement and its contrapositive always have the same truth value. Here both are false, as 2 and -2 show, but the question asks only for the form.

Answer: (3) If the squares of two numbers are equal, then the numbers are equal.

177 Answer: B

Ref 3049

Medium ~3 min Sets and their representation

QUICK APPROACH Use the distributive law to simplify $\sim s \vee (\sim r \wedge s)$ before negating.

Simplify the expression first, then negate it with De Morgan's law.

Step 1. Read the expression as $\sim s \vee (\sim r \wedge s)$.

Step 2. Distribute: $(\sim s \vee \sim r) \wedge (\sim s \vee s)$. Since $\sim s \vee s$ is always true, the expression is $\sim s \vee \sim r$.

Step 3. Negate: $\sim (\sim s \vee \sim r) = s \wedge r$ by De Morgan's law.

Step 4. Check with s and r both true: the original is $F \vee (F \wedge T) = F$, so its negation is true, and $s \wedge r$ is true. With s false, the original is true, so the negation is false, and $s \wedge r$ is false.

Answer: (B) $s \wedge r$

178 Answer: C

Ref 3089

Medium ~2 min Logic and Statements

QUICK APPROACH Use the distributive law on $p \vee (\sim p \wedge q)$ before negating.

Simplify the expression first, then negate it with De Morgan's law.

Step 1. Distribute: $p \vee (\sim p \wedge q) = (p \vee \sim p) \wedge (p \vee q)$.

Step 2. $p \vee \sim p$ is always true, so the expression is $p \vee q$.

Step 3. Negate: $\sim (p \vee q) = \sim p \wedge \sim q$.

Step 4. Check with p and q both false: the original is $F \vee (T \wedge F) = F$, so its negation is true, and $\sim p \wedge \sim q$ is true.

Step 5. So the negation is true only when both p and q are false. In words, the original says that at least one of p, q is true.

Answer: $(C) \sim p \wedge \sim q$

COMMON TRAPS

- **Forgetting to switch OR to AND.** De Morgan's law turns $\sim (A \vee B)$ into $\sim A \wedge \sim B$, not $\sim A \vee \sim B$. *Avoid it:* Change the connective and negate each part in one step.
- **Losing a double negation.** $\sim (\sim p \wedge q) = p \vee \sim q$, because $\sim (\sim p) = p$. *Avoid it:* Write every negation sign when expanding nested brackets.

REVISE De Morgan's laws with nested negations.

179 Answer: 3

Ref 3090

Medium ~2 min Relations

The implication $p \rightarrow (\sim q \vee r)$ (standard parsing, with $\vee$ binding tighter) is false only if antecedent p is T and consequent $\sim q \vee r$ is F.

$\sim q \vee r$ is F only when both $\sim q = F$ and $r = F$. $\sim q = F$ forces $q = T$. Hence the only solution is $p = T, q = T, r = F$.

Option [1] (T, F, T) encodes the negation error: mistakenly taking $\sim q = F$ to mean $q = F$ (instead of $q = T$) while setting $r = T$. That assignment actually yields $T \rightarrow (T \vee T) = T \rightarrow T = T$, not F.

Direct check: for T, T, F we have $\sim T = F, F \vee F = F$, so $T \rightarrow F = F$. All other options make the statement true.

Thus the correct choice is [3].

COMMON TRAPS

- **Copying the truth value of $\sim q$ onto q .** If $\sim q$ is false, then q is true, not false. *Avoid it:* Write q and $\sim q$ in two columns before reading off values.
- **Treating OR as false when one part is false.** A disjunction $\sim q \vee r$ is false only when both parts are false. *Avoid it:* For a false OR, set every part to false at once.

REVISE The single false row of an implication. · Truth tables of OR and NOT.

180 Answer: 4

Ref 3161

Medium ~3 min Sets and their representation

Check each statement using $A \cap B \subseteq C$ and $A \cap B \neq \phi$, and look for a counterexample to the doubtful one.

Step 1. Since $A \cap B \subseteq B$ and $A \cap B \subseteq C$, we get $A \cap B \subseteq B \cap C$. As $A \cap B \neq \phi$, also $B \cap C \neq \phi$. Statement (1) is true.

Step 2. By the distributive law, $(C \cup A) \cap (C \cup B) = C \cup (A \cap B)$. Since $A \cap B \subseteq C$, this is C . Statement (2) is true.

Step 3. Write $A = (A - B) \cup (A \cap B)$. If $A - B \subseteq C$, both parts lie in C , so $A \subseteq C$. Statement (3) is true.

Step 4. Take $A = \{1, 4\}, B = \{1\}, C = \{1, 4\}$. Then $A \cap B = \{1\} \subseteq C$ and $A - C = \phi \subseteq B$, but $4 \in A$ and $4 \notin B$. So statement (4) can fail.

Answer: (4) If $(A - C) \subseteq B$, then $A \subseteq B$

COMMON TRAPS

- **Treating option 2 as false instead of applying distributive identity.** $(C \cup A) \cap (C \cup B) = C \cup (A \cap B)$, and since $A \cap B \subseteq C$, this equals C . *Avoid it:* Expand the expression using $(X \cup Y) \cap (X \cup Z) = X \cup (Y \cap Z)$ before judging equality.
- **Assuming $A \cap C \subseteq B$ from $A \cap B \subseteq C$.** $A \cap B \subseteq C$ only says elements common to A and B lie in C ; it does not say elements common to A and C lie in B . *Avoid it:* Draw a Venn diagram or test with a counterexample where C contains extra elements of A not in B .

REVISE Set identities, especially distributive laws involving union and intersection. · Practice converting containments into element-wise implications.

181 Answer: A

Ref 3172

Easy ~1 min Relations

The logical implication $p \Rightarrow s$ (where $s \equiv q \vee r$) is false in exactly one case: when the antecedent p is true (T) and the consequent s is false (F). This is the decisive truth table row for $\rightarrow$. For $q \vee r$ to evaluate to F, both disjuncts must be F by the definition of inclusive OR; thus $q = F$ and $r = F$. Combining these yields $p = T, q = F, r = F$, matching option [A].

Option [C] (F, F, F) encodes the common mistake of assuming a false antecedent renders an implication false. In reality, $F \Rightarrow F$ is true, so when $p = F$ the compound statement cannot be false. Verifying all options confirms this: [A] gives $T \rightarrow (F \vee F) = T \rightarrow F = F$; [B] gives $F \rightarrow T = T$; [C] gives $F \rightarrow F = T$; [D] gives $T \rightarrow T = T$. Only [A] satisfies the given condition that the implication is false. This follows directly from the standard truth table for implication and disjunction without assuming any other connectives.

182 Answer: 2

Ref 3264

Medium ~3 min Logic

To solve the given logical statement $[\sim (\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$, follow these steps:

Step 1: Simplify $\sim (\sim p \vee q)$.

$$\sim (\sim p \vee q) = p \wedge \sim q$$

(Using De Morgan's laws)

Step 2: Substitute back into the original expression.

$$[(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$$

Step 3: Apply the distribution of the logical AND over the OR.

$$(p \wedge \sim q \wedge (\sim q \wedge r)) \vee (p \wedge r \wedge (\sim q \wedge r))$$

Step 4: Simplify both terms:

• First term:

$$(p \wedge \sim q \wedge r)$$

• Second term:

$$(p \wedge r \wedge \sim q)$$

Step 5: Combine the terms. Since both terms simplify to the same expression, use:

$$(p \wedge r) \wedge \sim q$$

Thus, the equivalent form is $(p \wedge r) \wedge \sim q$, which matches option 2.

COMMON TRAPS

- **Applying De Morgan's law without changing both the connector and the negations.** Correctly, $\sim (A \vee B) = \sim A \wedge \sim B$. Here $A = \sim p$, so $\sim A = p$, and $B = q$, so $\sim B = \sim q$. *Avoid it:* After De Morgan, do two checks: connector $\vee \rightarrow \wedge$, and every term inside gets negated, including the already-negated $\sim p$.
- **Dropping the outside conjunction with $(\sim q \wedge r)$.** The full statement is $[\dots] \wedge (\sim q \wedge r)$, so the final result must imply both $\sim q$ and r . *Avoid it:* Before matching options, verify that your answer enforces $\sim q$ and r because they are explicitly conjoined outside the bracket.

REVISE De Morgan's laws with double negation: $\sim (\sim p \vee q) = p \wedge \sim q$. Practice distribution in the exact form $(A \vee B) \wedge C = (A \wedge C) \vee (B \wedge C)$.

183 Answer: 4

Ref 3279

Medium ~3 min Sets and their representation

QUICK APPROACH Set the exponent equal to zero to find A , solve the inequality over the integers for B , then count $A \times B$.

A power of 2 equals 1 only when the exponent is 0.

Step 1. $(x+2)(x^2-5x+6) = (x+2)(x-2)(x-3) = 0$, so $A = \{-2, 2, 3\}$.

Step 2. $-3 < 2x-1 < 9$ gives $-1 < x < 5$, so $B = \{0, 1, 2, 3, 4\}$.

Step 3. $A \times B$ has $3 \times 5 = 15$ elements.

Step 4. A set with 15 elements has 2^{15} subsets.

Answer: $(4) 2^{15}$

COMMON TRAPS

- **Including endpoints.** The inequality $-1 < x < 5$ is strict, so -1 and 5 are not in B . *Avoid it:* List the integers strictly between the bounds.
- **Counting elements instead of subsets.** $A \times B$ has 15 elements, and the question asks for its number of subsets. *Avoid it:* Raise 2 to the number of elements.

REVISE Cartesian products and power sets.

184 Answer: B

Ref 3360

Medium ~3 min Boolean Algebra

Step 1: Consider the expression $((p \wedge q) \vee (p \vee \neg q)) \wedge (\neg p \wedge \neg q)$.

Step 2: Simplify the first part $(p \wedge q) \vee (p \vee \neg q)$.

- For $p = 1$, regardless of q , $(p \vee \neg q) = 1$.
- Hence, the expression simplifies to 1 if $p = 1$, and $\neg q$ if $p = 0$.

Step 3: Combine with $(\neg p \wedge \neg q)$.

- The expression $(\neg p \wedge \neg q)$ is true only when both $\neg p$ and $\neg q$ are true.
- Hence, the entire expression is equivalent to $(\neg p) \wedge (\neg q)$.

Step 4: Therefore, the expression simplifies to $(\neg p) \wedge (\neg q)$, which is option B.

185 Answer: 2

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Medium ~2 min Relations

QUICK APPROACH Replace $a \rightarrow b$ by $\sim a \vee b$ and then negate using De Morgan's law.

Rewrite the conditional as a disjunction, then apply De Morgan's law.

Step 1. For any statements, $a \rightarrow b$ is equivalent to $\sim a \vee b$.

Step 2. So $\sim p \rightarrow q$ is equivalent to $\sim (\sim p) \vee q = p \vee q$.

Step 3. Then $\sim (\sim p \rightarrow q)$ is equivalent to $\sim (p \vee q)$.

Step 4. By De Morgan's law, $\sim (p \vee q) = \sim p \wedge \sim q$.

Step 5. Check: the expression is true only when $\sim p \rightarrow q$ is false, that is when $\sim p$ is true and q is false. That is exactly $p \wedge \sim q$.

Answer: $(2) \sim p \wedge \sim q$

COMMON TRAPS

- **Wrong negation of a conditional.** $\sim (a \rightarrow b)$ is $a \wedge \sim b$; here a is $\sim p$, so the answer keeps $\sim p$. *Avoid it:* Substitute $a = \sim p$ carefully.
- **Dropping a double negation.** $\sim (\sim p)$ is p , which appears inside the disjunction before the final negation. *Avoid it:* Simplify one negation at a time.

REVISE Negation of conditional statements.



The full JEE Main: Mathematics has every chapter, chapter-wise and year-wise, in the Jeenius app.
